PM/00-39

Top quark production at future lepton colliders in the asymptotic regime

M. Beccaria[a,b], F.M. Renard[c] and C. Verzegnassi[d,e]

[a] *Dipartimento di Fisica, Università di Lecce*
Via Arnesano, 73100 Lecce, Italy.

[b] *INFN, Sezione di Lecce*
Via Arnesano, 73100 Lecce, Italy.

[c] *Physique Mathématique et Théorique, UMR 5825*
Université Montpellier II, F-34095 Montpellier Cedex 5.

[d] *Dipartimento di Fisica Teorica, Università di Trieste,*
Strada Costiera 14, Miramare (Trieste)

[e] *INFN, Sezione di Trieste*

Abstract

The production of a $t\bar{t}$ pair from lepton-antilepton annihilation is considered for values of the center of mass energy much larger than the top mass, typically of the few TeV size. In this regime a number of simplifications occurs that allows to derive the leading asymptotic terms of various observables using the same theoretical description that was used for light quark production. Explicit examples are shown for the Standard Model and the Minimal Supersymmetric Standard Model cases.

PACS numbers: 12.15.-y, 12.15.Lk, 14.65.Ha, 14.80.Ly

I. INTRODUCTION.

In a number of recent papers [1–3], the production of lepton-antilepton and quark-antiquark pairs from lepton-antilepton colliders was considered at the one-loop level, with special emplasis on the "asymptotic" leading behaviour of various observables, in the case of "light" (i.e. u, d, s, c, b) quarks. This analysis was performed both for the Standard Model (SM) and for the Minimal Supersymmetric Standard Model (MSSM) cases, and the results are fully illustrated in Refs. [1], [2], [3]. In particular, it was stressed that the leading asymptotic behaviour is not provided by the known Renormalization Group (RG) logarithms alone, but from the overall term which is obtained adding to the RG linear logarithms those of the so-called "Sudakov-type" [4]. The latter ones are both of quadratic and of linear type in the SM, but only of linear type in the SUSY additional contributions that appear in the extra relevant one-loop diagrams. In the case of final bottom-antibottom production, it was stressed that important linear logarithmic contributions that are also proportional to the squared top mass (and, for SUSY diagrams, also to the squared bottom mass) cannot be neglected, and their numerical effect was illustrated in several Figures of Ref. [3], where special emphasis was given to the "asymptotic" energy region between 3 and 5 TeV, which is supposed to be covered by the future CERN CLIC accelerator [5] and, possibly, by a future muon collider [6].

A very useful ingredient that was used in the theoretical analysis of Refs. [1], [2], [3] is the possibility of exploiting an approach in which several gauge-invariant combinations of one-loop quantities (self-energies, vertices and boxes) are "subtracted" at Z-peak. This introduces as theoretical input quantities (widths, asymmetries) that have been measured with extreme precision at LEP1, SLC. The reward is that of decreasing systematically the number of theoretical parameters that appear in theoretical models beyond the SM and of leaving, as one-loop functions, quantities that are from the begining gauge-invariant and finite [7], as illustrated in previous references [8]. In principle, this approach can only be used for final fermion-antifermion pairs that can be physically produced at the Z peak. Due to this apparently rigid criterium, it was not applied until now to the case of final top-antitop production.

The aim of this preliminary paper is that of showing that, if one only considers the "asymptotic" few TeV regime, it is possible to treat top-antitop production by the same theoretical approach that was used to describe the production of bottom-antibottom with the only formal replacement, as a theoretical input, of the (forbidden) Z decay width into top-antitop with the available Z decay into charm-anticharm. The only residual theoretical difference is that asymptotic regime description will be provided by known and calculated one-loop vertices containing the squared top (and bottom) mass. This will allow to provide theoretical prediction for several observables of the process, in the few TeV regime, for both the SM and the MSSM cases, thus completing the already available treatment given for "light" fermion production in previous references.

Technically speaking, the paper will be organised as follows: Section II will contain a brief kinematical description of the process and of its simplifications in the asymptotic regime; in Section III the relevant one-loop diagrams giving rise to the leading asymptotic contributions will be given for the separate SM and MSSM cases. Section IV will contain the numerical predictions for various observables, and finally a short conclusive discussion is

made in Section V. The asymptotic expressions of the relevant quantities that determine the observables of the process are given in Appendix A; the definitions of the helicity amplitudes and of the observables specific to the final $t\bar{t}$ state can be found in Appendix B.

II. GENERAL DESCRIPTION OF $t\bar{t}$ PRODUCTION FROM l^+l^- ANNIHILATION

In full generality, $t\bar{t}$ production from l^+l^- annihilation at one-loop differs from light fermion production because two new structures appear in the theoretical description that are a consequence of the not negligible top mass. This can be visualized in two equivalent ways, either by considering a "conventional" formalism (like that used in the light quark case) or by introducing the helicity amplitudes [9], that are now experimentally more meaningful as the final top polarization can be measured. To understand the origin of the extra structures, it will be sufficient to consider the theoretical expansion of a one loop vertex like that represented in Fig.1 or 2, with either a photon or a Z entering the bubble. In full generality, with CP-conserving interactions one can associate to that diagram the quantity:

$$\Gamma^X_\mu = -e^X[\gamma_\mu(g^X_{Vt} - g^X_{At}\gamma^5) + \frac{d^X}{m_t}(p-p')_\mu] \tag{2.1}$$

where $X = \gamma, Z$, $e^\gamma = |e|$, $e^Z = \frac{|e|}{2s_W c_W}$ and p, p' represent the outgoing t, $\bar{t}$ momenta; g^X_{Vt}, g^X_{At}, d^X are $O(\alpha)$ one-loop contributions which in general are $q^2 = (p+p')^2$ dependent. The two new quantities d^X enter because the top mass cannot now be neglected and appear in the various theoretical expressions at one loop, making the overall number of independent amplitudes of the process to increase from four (in massless fermion production) to six. This is because the <u>three</u> independent coefficients of eq.(2.1) will be combined with the <u>two</u> independent coefficients (g^X_{Vl}, g^X_{Al}) of the initial (massless) lepton vertex.

Starting from this general statement, it is now relatively easy to provide the expressions that appear at one loop e.g. in the helicity amplitudes formalism. This procedure, which would be essential for a general description at "moderate" c.m. energies, will be fully developed in a dedicated forthcoming paper [9]. But for the specific purposes of an "asymptotic" energy description, there will be a welcome simplification. In fact, it is possible to see immediately from the structure of the one-loop Feynman diagrams that, <u>in the specific cases</u> of the SM and in the MSSM, the coefficients of the new extra Lorentz structure $(p-p')^\mu$ vanish at large q^2 like $1/q^2$, while those of the "conventional" Lorentz structures (γ^μ, $\gamma^\mu\gamma^5$) can produce either quadratic or linear logarithms. Therefore, the leading terms of $t\bar{t}$ production at "asymptotic" energies are exactly those that would be computed in a 'conventional" scheme in which the new "scalar" component of eq.(2.1) has been neglected, and <u>four</u> independent gauge-invariant combinations survive that are, formally, equivalent to those of the final light quark case.

At the one loop level, this means that it will be possible to write the asymptotic invariant scattering amplitude of the process $l^+l^- \to t\bar{t}$ in the following way:

$$A^{(1)(V,A\ only)}_{lt}(q^2,\theta) = j^{\mu(\gamma)}_t[\frac{1}{q^2}(1-\tilde{F}^\gamma_{lt}(q^2,\theta))]j^{(\gamma)}_{\mu,l} + j^{\mu(Z)}_t[\frac{1}{q^2-m^2_Z}(1-\frac{\tilde{A}^Z_{lt}(q^2,\theta)}{q^2-m^2_Z})]j^{(Z)}_{\mu,l}$$
$$-j^{\mu(Z)}_t[\frac{1}{q^2-m^2_Z}\frac{\tilde{A}^{\gamma Z}_{lt}(q^2,\theta)}{q^2}]j^{(\gamma)}_{\mu,l} - j^{\mu(\gamma)}_t[\frac{1}{q^2-m^2_Z}\frac{\tilde{A}^{Z\gamma}_{lt}(q^2,\theta)}{q^2}]j^{(Z)}_{\mu,l} \tag{2.2}$$

where $j^{\mu(\gamma,Z)}_{l,t}$ are the conventional Lorentz structures used as a basis for the decomposition of the general amplitudes [7]:

$$j^{(\gamma)}_{\mu,f} = -|e|Q_f\gamma_\mu \tag{2.3}$$

$$j^{(Z)}_{\mu,f} = -\frac{|e|}{2s_W c_W}\gamma_\mu(g^0_{V,f} - g^0_{A,f}\gamma^5) \tag{2.4}$$

with $g^0_{V,f} = I^3_f v_f$, $g^0_{A,f} = I^3_f$, $v_f = 1 - 4|Q_f|s^2_W$.

The four quantities $\tilde{F}^{\gamma}_{lt}(q^2,\theta)$, $\tilde{A}^{Z}_{lt}(q^2,\theta)$, $\tilde{A}^{\gamma Z}_{lt}(q^2,\theta)$ and $\tilde{A}^{Z\gamma}_{lt}(q^2,\theta)$ are the generalized photon and Z one-loop self-energies, defined in [7], which contain also vertex and box contributions [10]. Starting from these quantities one will now generalize for $f = t$ the treatment which was done in [7] in order to construct the four "subtracted" gauge-invariant functions $\widetilde{\Delta}_{\alpha,lf}(q^2,\theta)$, $R_{lf}(q^2,\theta)$, $V^{\gamma Z}_{lf}(q^2,\theta)$ and $V^{Z\gamma}_{lf}(q^2,\theta)$. In fact for the photon component $\widetilde{\Delta}_{\alpha,lt}(q^2,\theta)$ the subtraction is still performed at $q^2 = 0$, leaving as theoretical input $\alpha(0)$ and one-loop quantities that will depend also on m^2_t (and, in principle, on m^2_b).

$$\widetilde{\Delta}_{\alpha,lt}(q^2,\theta) = \widetilde{F}^{\gamma}_{lt}(0,\theta) - \widetilde{F}^{\gamma}_{lt}(q^2,\theta) \tag{2.5}$$

with

$$\tilde{F}^{\gamma}_{lt}(q^2,\theta) = F^{\gamma}_{lt}(q^2) - (\Gamma^{(\gamma)}_{\mu,l}, j^{(\gamma)}_{\mu,l}) - (\Gamma^{(\gamma)}_{\mu,t}, j^{(\gamma)}_{\mu,t}) - q^2 A^{(Box)}_{\gamma\gamma,lt}(q^2,\theta) \tag{2.6}$$

where the notation (Γ, j) means the projection of the contribution to the initial or to the final vertex Γ on the element j of the basis, see eq.(2.3,2.4) and $A^{(Box)}_{\gamma\gamma,lt}(q^2,\theta)$ is the projection of the $l^+l^- \to t\bar{t}$ box contribution on the element $j^{(\gamma)}_t j^{(\gamma)}_l$ of the basis, [7].

To illustrate the treatment of the 3 remaining quantities, we consider briefly the case of $\tilde{A}^{(Z)}_{lt}(q^2,\theta)$ and write its theoretical expression adding and subtracting the analogous quantity $\tilde{A}^{(Z)}_{lc}(q^2,\theta)$. After straightforward manipulations, this will lead to the following situation:
a) The theoretical input which will appear in the "Born" term will be identical with that of charm-anticharm production, in the sense that it will contain the partial width of Z into $c\bar{c}$, exactly like in Ref. [7] for $f = c$.
b) The residual one-loop quantity will be the difference between the non universal vertices and boxes of top production and the corresponding quantities of charm production. In the notation of Ref. [7], this will correspond to the introduction of a "modified" gauge-invariant $\hat{R}_{lt}$ function defined in terms of the generalized ZZ self-energy as:

$$\begin{aligned}\hat{R}_{lt}(q^2,\theta) = R_{lc}(q^2,\theta) &- (\Gamma^{(Z)}_{\mu,t}(q^2) - \Gamma^{(Z)}_{\mu,c}(q^2), j^{(Z)}_{\mu,t}) \\ &-(q^2 - m^2_Z)[A^{box}_{ZZ,lt}(q^2,\theta) - A^{box}_{ZZ,lc}(q^2,\theta)]\end{aligned} \tag{2.7}$$

with the c-quark function

$$R_{lc}(q^2,\theta) = I_{Z,lc}(q^2,\theta) - I_{Z,lc}(M^2_Z,\theta) \tag{2.8}$$

involving the generalized ZZ function for the production of a $c\bar{c}$ pair

$$I_{Z,lc}(q^2,\theta) = \frac{q^2}{q^2 - M_Z^2}[\widetilde{F}^Z_{lc}(q^2,\theta) - \widetilde{F}^Z_{lc}(M_Z^2,\theta)] \tag{2.9}$$

A quite analogous procedure can be used for the interference terms $V^{\gamma Z}$ and $V^{Z\gamma}$. Without entering the full details, this leads to the introduction in the theoretical input of forward-backward asymmetry of the Z decay into charm-anticharm and to the introduction of a new gauge-invariant function:

$$\begin{aligned}\hat{V}^{Z\gamma}_{lt}(q^2,\theta) = V^{Z\gamma}_{lc}(q^2,\theta) - (\Gamma^{(Z)}_{\mu,t}(q^2) - \Gamma^{(Z)}_{\mu,c}(q^2), j^{(\gamma)}_{\mu,t})\\ -(q^2 - m_Z^2)[A^{box}_{Z\gamma,lt}(q^2,\theta) - A^{box}_{Z\gamma,lc}(q^2,\theta)]\end{aligned} \tag{2.10}$$

with

$$V^{Z\gamma}_{lc}(q^2,\theta) = \frac{\widetilde{A}^{Z\gamma}_{lc}(q^2,\theta)}{q^2} - \frac{\widetilde{A}^{Z\gamma}_{lc}(M_Z^2,\theta)}{M_Z^2} \tag{2.11}$$

$$\begin{aligned}\frac{\widetilde{A}^{Z\gamma}_{lc}(q^2,\theta)}{q^2} = \frac{A^{\gamma Z}(q^2)}{q^2} - \frac{q^2 - M_Z^2}{q^2}(\Gamma^{(\gamma)}_{\mu,l}(q^2), j^{(Z)}_{\mu,l})\\ -(\Gamma^{(Z)}_{\mu,c}(q^2), j^{(\gamma)}_{\mu,c}) - (q^2 - M_Z^2)A^{(Box)}_{Z\gamma,lc}(q^2,\theta)\end{aligned} \tag{2.12}$$

The other interference term $V^{\gamma Z}$ does not require specific tricks, as in the case of the photon component $\tilde{\Delta}_{\alpha,lt}(q^2,\theta)$. Its theoretical expression can be written in fact as :

$$V^{\gamma Z}_{lt}(q^2,\theta) = V^{\gamma Z}_{lc}(q^2,\theta) = \frac{\widetilde{A}^{\gamma Z}_{lt}(q^2,\theta)}{q^2} - \frac{\widetilde{A}^{\gamma Z}_{lt}(M_Z^2,\theta)}{M_Z^2} \tag{2.13}$$

with

$$\begin{aligned}\frac{\widetilde{A}^{\gamma Z}_{lt}(q^2,\theta)}{q^2} = \frac{A^{\gamma Z}(q^2)}{q^2} - (\frac{q^2 - M_Z^2}{q^2})(\Gamma^{(\gamma)}_{\mu,t}(q^2), j^{(Z)}_{\mu,t})\\ -(\Gamma^{(Z)}_{\mu,l}(q^2), j^{(\gamma)}_{\mu,l}) - (q^2 - M_Z^2)A^{(Box)}_{\gamma Z,lt}(q^2,\theta)\end{aligned} \tag{2.14}$$

and in eq.(2.14) no final (top) vertices or boxes appear at m_Z^2.

The previous equations that we wrote are quite general. From their expressions one can now determine in a straightforward way the related asymptotic behaviours. To obtain the latter ones, it will be sufficient to add to the "light" c quark functions, already computed in Refs. [1], [3] the extra non universal terms coming from the difference between top vertices and charm vertices (the difference between boxes will vanish asymptotically since the latter ones are not producing massive terms, as already illustrated in those references). This means that the only extra quantities to be computed for the specific purposes of this paper are the asymptotic "Sudakov-type" linear logaritms proportional to the squared top and bottom masses coming from the final top vertex. Their expressions will be given in the following Section III.

III. MASSIVE ONE-LOOP CONTRIBUTIONS FROM FINAL TOP VERTICES

For the specific purposes of this paper we shall only be interested in those contributions to the process of $t\bar{t}$ production that come from final top vertices and are proportional either to the squared top mass or (in practice, only for the SUSY case) to the squared bottom mass. These are coming from the diagrams shown in Fig.(1) for the SM and in Fig.(2) for the extra SUSY component of the MSSM, that we consider separately in this paper. Using our conventional definitions [1] of the one-loop vertex $i\Gamma_\mu$, we derive the components of the leading asymptotic behaviour that are proportional to the quark masses following the same procedure that was used in Ref. [3]. The results are given by the following equations:

$$\Gamma^{\gamma}_{\mu}(\mathrm{SM},\ \mathrm{massive}) \to \frac{e\alpha}{24\pi M_W^2 s_W^2}\ \ln q^2 \{m_t^2[(\gamma_\mu P_L)+2(\gamma_\mu P_R)]+m_b^2(\gamma_\mu P_L)\} \tag{3.1}$$

$$\begin{aligned}\Gamma^{Z}_{\mu}(\mathrm{SM},\ \mathrm{massive}) \to \frac{e\alpha}{96\pi M_W^2 s_W^3 c_W}\ & \ln q^2 \{(3-4s_W^2)m_t^2(\gamma_\mu P_L)-8s_W^2 m_t^2(\gamma_\mu P_R) \\ &+(3-4s_W^2)m_b^2(\gamma_\mu P_L)\}\end{aligned} \tag{3.2}$$

$$\begin{aligned}\Gamma^{\gamma}_{\mu}(\chi,\ \mathrm{massive}) \to \frac{e\alpha}{24\pi M_W^2 s_W^2}\ & \ln q^2 \{m_t^2(1+\cot^2\beta)[(\gamma_\mu P_L)+2(\gamma_\mu P_R)] \\ &+m_b^2(1+\tan^2\beta)(\gamma_\mu P_L)\}\end{aligned} \tag{3.3}$$

$$\begin{aligned}\Gamma^{Z}_{\mu}(\chi,\ \mathrm{massive}) \to \frac{e\alpha}{96\pi M_W^2 s_W^3 c_W}\ & \ln q^2 \{(3-4s_W^2)m_t^2(1+\cot^2\beta)(\gamma_\mu P_L) \\ &-8s_W^2 m_t^2(1+\cot^2\beta)(\gamma_\mu P_R)+(3-4s_W^2)m_b^2(1+\tan^2\beta)(\gamma_\mu P_L)\}\end{aligned} \tag{3.4}$$

$$\begin{aligned}\Gamma^{\gamma}_{\mu}(H) \to \frac{e\alpha}{24\pi M_W^2 s_W^2}\ & \ln q^2 \{m_t^2(\cot^2\beta)[(\gamma_\mu P_L)+2(\gamma_\mu P_R)] \\ &+m_b^2(\tan^2\beta)(\gamma_\mu P_L)\}\end{aligned} \tag{3.5}$$

$$\begin{aligned}\Gamma^{Z}_{\mu}(H) \to \frac{e\alpha}{96\pi M_W^2 s_W^3 c_W}\ & \ln q^2 \{(3-4s_W^2)m_t^2(\cot^2\beta)(\gamma_\mu P_L) \\ &-8s_W^2 m_t^2(\cot^2\beta)(\gamma_\mu P_R)+(3-4s_W^2)m_b^2(\tan^2\beta)(\gamma_\mu P_L)\}\end{aligned} \tag{3.6}$$

where $P_{L,R}=(1\mp\gamma^5)/2$. (H) denotes the contribution from the SUSY charged and neutral Higgses of the MSSM (the contribution from the SM Higgs has been subtracted) and (χ) denotes the contribution from charginos and neutralinos of the model.

Eqs.(3.1-3.6) are the new results of this paper. Note that the total MSSM massive contributions just correspond to the SM ones with the m_t^2 terms being multiplied by $2(1+\cot^2\beta)$ and the m_b^2 terms by $2(1+\tan^2\beta)$, a rule which had already been observed in Ref. [3]:

$$\begin{aligned}\Gamma^{\gamma}_{\mu}(\mathrm{MSSM},\ \mathrm{massive}) \to \frac{e\alpha}{12\pi M_W^2 s_W^2}\ & \ln q^2 \{m_t^2(1+\cot^2\beta)[(\gamma_\mu P_L)+2(\gamma_\mu P_R)] \\ &+m_b^2(1+\tan^2\beta)(\gamma_\mu P_L)\}\end{aligned} \tag{3.7}$$

$$\Gamma^Z_\mu(\mathrm{MSSM,\ massive}) \to \frac{e\alpha}{48\pi M_W^2 s_W^3 c_W}\ \ln q^2 \{(3-4s_W^2)m_t^2(1+\cot^2\beta)(\gamma_\mu P_L)$$
$$-8s_W^2 m_t^2(1+\cot^2\beta)(\gamma_\mu P_R) + (3-4s_W^2)m_b^2(1+\tan^2\beta)(\gamma_\mu P_L)\} \quad (3.8)$$

It should be stressed that, as already remarked in ref. [3], in the leading asymptotic SUSY "Sudakov" logarithms all the details of the MSSM, in particular the mixing parameters appearing in the chargino and neutralino mass matrices, are washed out in the large q^2 limit, and are "reshuffled" in subleading constant terms. The only supersymmetric parameter of the model that survives asymptotically is $\tan\beta$. This will affect the observables of top-antitop production in a potentially interesting way that will be fully examined in the final discussion.

To obtain the expressions of the various observables of the process $l^+l^- \to t\bar{t}$, one has to add the above massive contributions to those coming from the massless quark situation (in our case, that corresponding to charm production). The overall terms are incorporated into the four gauge-invariant quantities $\tilde{\Delta}_{\alpha,lt}$, $\hat{R}_{lt}$, $V_{lt}^{\gamma Z}$, $\hat{V}_{lt}^{Z\gamma}$ from which all the observables at one loop can be built in the asymptotic regime of the $t\bar{t}$ production process. We have listed the various contributions in the Appendix A, in the following order; first, the SM contributions, universal (RG) terms, non-universal massless terms and non-universal massive (m_t^2 and m_b^2 dependent) terms; secondly, the additional SUSY contributions split into the same three groups.

Starting from the formulae of Appendices A and B it is now possible to derive the theoretical predictions for the leading asymptotic behaviour of the observables of the process. These are exhibited in the following final Section IV.

IV. ASYMPTOTIC BEHAVIOUR OF THE OBSERVABLES OF $t\bar{t}$ PRODUCTION

For the process $l^+l^- \to t\bar{t}$ we can consider two sets of observables. First, we have computed the one-loop effects for the same set previously considered in the case of light quarks: the integrated $e^+e^- \to t\bar{t}$ cross section denoted by σ_t, the forward backward asymmetry $A_{FB,t}$, the longitudinal polarization asymmetry $A_{LR,t}$ and its forward-backward polarization asymmetry [11] A_t. Secondly, we have considered a new set of observables constructed from the spin density matrix of the final top [12]; the averaged top helicity H_t, its forward-backward asymmetry $H_{t,FB}$, as well as the same two observables H_t^{LR}, $H_{t,FB}^{LR}$ in the case of longitudinaly polarized $l^\pm$ beams. Their definition in terms of the top spin density matrix as well as the general expression of the helicity amplitudes are given in Appendix B; more details will be found in [9]. In general there could be 4 other observables related to the transverse top quark polarization, but for asymptotic energies the transverse polarization degree vanishes like $m_t/\sqrt{q^2}$.

The results for the asymptotic behaviour of each observables are given by the following equations with the various terms grouped in the following order: first in SM, the RG with the mass scale μ, followed by the linear and quadratic Sudakov (W diagrams) terms, the linear and quadratic Sudakov (Z diagrams) terms and finally the linear Sudakov term arising from the quadratic m_t^2 contribution; then, in bold face, the SUSY contributions, first the RG (SUSY) term with the mass scale μ, then the linear Sudakov (SUSY) m_t- and m_b-

independent term (scaled by the common mass M), the linear Sudakov (SUSY) term arising from the quadratic m_t^2 contribution (scaled by a common mass M') and in curly brackets the same term to which the $m_b^2\tan^2\beta$ contribution is added successively for $\tan\beta = 10$ and for $\tan\beta = 40$. This was done in order to show precisely the origin of the difference between the total SM prediction and the total SUSY part.We have chosen to use for simplicity common mass scales M and M' because of the present ignorance of the physical masses of the charginos, neutralinos and sfermions appearing in the triangle diagrams of Fig.2 as well as those of the charged and neutral Higgses appearing together with the top and the bottom quark. A change of reference scale is equivalent to the addition of an asymptotically negligible constant term; see Ref. [3] and [9] for a discussion of this point.

We first consider the four observables constructed from the differential cross section without measuring the final top quark polarization. In the following equations the various "subtracted" Born terms O^B are defined in terms of the Z-peak inputs as explained in Section II.

$$\begin{aligned}\sigma_t = \sigma_t^B\{1 + &\frac{\alpha}{4\pi}\{(8.87N - 33.16)\ln\frac{q^2}{\mu^2} + (22.79\ln\frac{q^2}{M_W^2} - 5.53\ln^2\frac{q^2}{M_W^2})\\ &+(3.52\ln\frac{q^2}{M_Z^2} - 1.67\ln^2\frac{q^2}{M_Z^2}) - \ 14.21\ln\frac{q^2}{m_t^2}\\ &+(\mathbf{4.44\,N + 11.09})\ \ln\frac{\mathbf{q^2}}{\boldsymbol{\mu}^\mathbf{2}} - \mathbf{10.09}\ \ln\frac{\mathbf{q^2}}{\mathbf{M^2}} - \mathbf{42.63\{-15.33\}\{-27.48\}}\ln\frac{\mathbf{q^2}}{\mathbf{M'^2}}\}\end{aligned} \tag{4.1}$$

$$\sigma_t^B = 0.182\ \mathrm{pb}/q^2(\mathrm{TeV}^2) \tag{4.2}$$

$$\begin{aligned}A_{FB,t} = A_{FB,t}^B + &\frac{\alpha}{4\pi}\{(0.45N - 4.85)\ln\frac{q^2}{\mu^2} - (1.79\ln\frac{q^2}{M_W^2} + 0.17\ln^2\frac{q^2}{M_W^2})\\ &-(1.26\ln\frac{q^2}{M_Z^2} + 0.06\ln^2\frac{q^2}{M_Z^2}) + 0.61\ln\frac{q^2}{m_t^2}\\ &+(\mathbf{0.22\,N + 1.29})\ \ln\frac{\mathbf{q^2}}{\boldsymbol{\mu}^\mathbf{2}} - \mathbf{0.23}\ \ln\frac{\mathbf{q^2}}{\mathbf{M^2}} + \mathbf{1.83\{0.54\}\{-0.68\}}\ln\frac{\mathbf{q^2}}{\mathbf{M'^2}}\}\ .\end{aligned} \tag{4.3}$$

$$A_{FB,t}^B = 0.607 \tag{4.4}$$

$$\begin{aligned}A_{LR,t} = A_{LR,t}^B + &\frac{\alpha}{4\pi}\{(2.06N - 22.43)\ln\frac{q^2}{\mu^2} + (14.75\ln\frac{q^2}{M_W^2} - 3.54\ln^2\frac{q^2}{M_W^2})\\ &+(0.40\ln\frac{q^2}{M_Z^2} - 0.49\ln^2\frac{q^2}{M_Z^2}) + 3.79\ln\frac{q^2}{m_t^2}\\ &+(\mathbf{1.03\,N + 5.95})\ \ln\frac{\mathbf{q^2}}{\boldsymbol{\mu}^\mathbf{2}} - \mathbf{4.03}\ \ln\frac{\mathbf{q^2}}{\mathbf{M^2}} + \mathbf{11.36\{3.36\}\{-4.23\}}\ln\frac{\mathbf{q^2}}{\mathbf{M'^2}}\},\end{aligned} \tag{4.5}$$

$$A_{LR,t}^B = 0.336 \tag{4.6}$$

$$
\begin{aligned}
A_t = A_t^B &+ \frac{\alpha}{4\pi}\{(1.82N-19.77)\ln\frac{q^2}{\mu^2} + (8.68\ln\frac{q^2}{M_W^2} - 2.76\ln^2\frac{q^2}{M_W^2}) \\
&+(0.09\ln\frac{q^2}{M_Z^2} - 0.45\ln^2\frac{q^2}{M_Z^2}) + 3.69\ln\frac{q^2}{m_t^2} \\
&+(\mathbf{0.91\,N+5.25})\ \ln\frac{\mathbf{q^2}}{\mu^{\mathbf{2}}} - \mathbf{3.20}\ \ln\frac{\mathbf{q^2}}{\mathbf{M^2}} + \mathbf{11.06\{3.27\}\{-4.11\}}\ln\frac{\mathbf{q^2}}{\mathbf{M'^2}}\},
\end{aligned}
\tag{4.7}
$$

$$
A_t^B = 0.164 \tag{4.8}
$$

We have then considered the four other observables constructed from the final top quark helicity and defined in Appendix B. In the asymptotic regime (i.e neglecting m_t^2/q^2 terms and new Lorentz structures) we notice that, if there were no Box contributions introducing extra θ-dependences, these four observables would be exactly related to the four previous ones. This is obvious from the fact that, in this limit, there are only four independent combinations of photon and Z coupling which describe the differential cross section for any top or antitop helicity (denoted G_1, G_2, G_4 and G_5 in ref. [13]). The relations would be the following ones:

$$
H_t^{no\ Box} \equiv -\ \frac{4}{3}\ A_t^{no\ Box} \tag{4.9}
$$

$$
H_{t,FB}^{no\ Box} \equiv -\ \frac{3}{4}\ A_{LR,t}^{no\ Box} \tag{4.10}
$$

$$
H_t^{LR,no\ Box} \equiv -\ \frac{4}{3}\ A_{FB,t}^{no\ Box} \tag{4.11}
$$

$$
H_{t,FB}^{LR,no\ Box} \equiv -\ \frac{3}{4} \tag{4.12}
$$

At non asymptotic energies relations (4.10) and (4.12), contrarily to (4.9) and (4.11), should be affected not only by Box effects, but also by m_t^2/q^2 terms and by contributions from the new Lorentz structures; more details will be given in [9].

Taking the SM box contributions into account we obtain to the following results:

$$
\begin{aligned}
H_t = H_t^B &+ \frac{\alpha}{4\pi}\{(-2.42N+26.36)\ln\frac{q^2}{\mu^2} + (-15.90\ln\frac{q^2}{M_W^2} + 3.67\ln^2\frac{q^2}{M_W^2}) \\
&+(-0.74\ln\frac{q^2}{M_Z^2} + 0.59\ln^2\frac{q^2}{M_Z^2}) - 4.91\ln\frac{q^2}{m_t^2} \\
&+(\mathbf{-1.21\,N-7.00})\ \ln\frac{\mathbf{q^2}}{\mu^{\mathbf{2}}} + \mathbf{4.27}\ \ln\frac{\mathbf{q^2}}{\mathbf{M^2}} - \mathbf{14.74\{-4.36\}\{+5.49\}}\ln\frac{\mathbf{q^2}}{\mathbf{M'^2}}\},
\end{aligned}
\tag{4.13}
$$

$$
H_t^B = -0.219 \tag{4.14}
$$

$$H_{t,FB} = H^B_{t,FB} + \frac{\alpha}{4\pi}\{(-1.55N + 16.81)\ln\frac{q^2}{\mu^2} + (-7.82\ln\frac{q^2}{M_W^2} + 2.65\ln^2\frac{q^2}{M_W^2})$$
$$+(0.23\ln\frac{q^2}{M_Z^2} + 0.37\ln^2\frac{q^2}{M_Z^2}) - 2.84\ln\frac{q^2}{m_t^2}$$
$$+(-\mathbf{0.77\,N} - \mathbf{4.46})\ \ln\frac{\mathbf{q^2}}{\mu^{\mathbf{2}}} + \mathbf{3.02}\ \ln\frac{\mathbf{q^2}}{\mathbf{M^2}} - \mathbf{8.52}\{-\mathbf{2.52}\}\{+\mathbf{3.17}\}\ln\frac{\mathbf{q^2}}{\mathbf{M'^2}}\}, \quad (4.15)$$

$$H^B_{t,FB} = -0.252 \quad (4.16)$$

$$H^{LR}_t = H^{LR,B}_t + \frac{\alpha}{4\pi}\{(-0.59N + 6.46)\ln\frac{q^2}{\mu^2} + (-1.91\ln\frac{q^2}{M_W^2} + 0.23\ln^2\frac{q^2}{M_W^2})$$
$$+(0.70\ln\frac{q^2}{M_Z^2} + 0.08\ln^2\frac{q^2}{M_Z^2}) - 0.81\ln\frac{q^2}{m_t^2}$$
$$+(-\mathbf{0.30\,N} - \mathbf{1.71})\ \ln\frac{\mathbf{q^2}}{\mu^{\mathbf{2}}} + \mathbf{0.31}\ \ln\frac{\mathbf{q^2}}{\mathbf{M^2}} - \mathbf{2.44}\{-\mathbf{0.72}\}\{+\mathbf{0.91}\}\ln\frac{\mathbf{q^2}}{\mathbf{M'^2}}\}, \quad (4.17)$$

$$H^{LR,B}_t = -0.809 \quad (4.18)$$

$$H^{LR}_{t,FB} = H^{LR,B}_{t,FB} + \frac{\alpha}{4\pi}\{(0.)\ln\frac{q^2}{\mu^2} + (3.24\ln\frac{q^2}{M_W^2} + (0.)\ln^2\frac{q^2}{M_W^2})$$
$$+(0.50\ln\frac{q^2}{M_Z^2} + (0.)\ln^2\frac{q^2}{M_Z^2}) + (0.)\ln\frac{q^2}{m_t^2}$$
$$+(\mathbf{0}.)\ \ln\frac{\mathbf{q^2}}{\mu^{\mathbf{2}}} + (\mathbf{0}.)\ \ln\frac{\mathbf{q^2}}{\mathbf{M^2}} + (\mathbf{0}.)\ln\frac{\mathbf{q^2}}{\mathbf{M'^2}}\}, \quad (4.19)$$

$$H^{LR,B}_{t,FB} = -0.750 \quad (4.20)$$

One can check that, indeed apart from the two SM coefficients of $\ln\frac{q^2}{M_W^2}$ and $\ln\frac{q^2}{M_Z^2}$, affected by the Box contributions, all other coefficients, as well as the "Born" terms satisfy the relations (4.9)-(4.12). In particular eq,(4.12) is responsible for the appearence of the various zeros in eq.(4.19). So the physical content of the four observables constructed with the top helicity is almost the same as that of the polarized differential cross section. The importance of the angular dependence in the SM box contribution can be appreciated from the size of the non zero coefficients in eq.(4.19) and will be illustrated in a figure given below. A comparison with experimental data on top quark polarization should be useful for a confirmation of the results obtained from the cross section and the asymmetries and should constitute a check of the model (SM or MSSM) and of the absence of unexpectedly large asymptotic contributions.

Eqs.(4.1)-(4.20) are the main result of this paper. To better appreciate their message, we have plotted in the following Figs.(3)-(6) and (7)-(10), the asymptotic terms, with the

following convention: for the cross section, we show the relative effect; for asymmetries and helicities, the absolute effect. To fix a scale, we also write in the Figure captions the value of the (asymptotic) "Born " terms and we have put $\mu = M_Z$ for the RG terms and $M = M' = m_t$ for the SUSY terms. The plots have been drawn in an energy region between one and ten TeV.

We have plotted the overall SM value, the overall MSSM value for purposes of comparison, and the approximate expressions that would be obtained by only retaining the asymptotic RG logaritms in both cases. From inspection of these Figures a number of conclusions can be drawn. They are listed in the final Section V.

V. CONCLUSIONS

In this paper we have extended to the case of the process $e^+e^- \to t\bar{t}$ the study of the high energy behaviour of four-fermion processes that we had undertaken in previous works for the case of light fermions. First we have shown how the Z-peak subtracted representation can still be used to describe this process, by taking as inputs the measurements of the charm-anticharm process at the Z-peak, and by putting inside the subtracted functions the difference between the $t\bar{t}$ and the $c\bar{c}$ one loop effects. We have then applied this method in order to obtain well-defined predictions for the high energy behaviour of the various observables that can be experimentally studied in $e^+e^- \to t\bar{t}$. We have made illustrations with the one loop effects that appear in the SM and in the MSSM. The results of this investigation are now summarized.

1) The leading electroweak effect at the one loop level is quite sizeable in the TeV region in all observables, with the only (expected) exception of the forward-backward asymmetry, where the squared Sudakov logarithms are practically vanishing, as a consequence of a general rule already discussed in Ref. [3]. The effect increases with energy, following a trend that is drastically different from that of the smooth and much smaller pure RG approximation, and it appears therefore to be essentially governed by the various logarithms of "Sudakov-type".

2) The leading effects for top production are systematically larger than those in the corresponding lepton or "light" (u, d, s, c, b) quark production observables. This is valid both in the SM and in the MSSM situation. In the latter case, top production exhibits also in the leading terms a drastic dependence on $\tan\beta$, much stronger than that of bottom production (shown in Ref. [3]).

3) The validity of a one loop perturbative expansion for top production seems to us neither too likely nor too unlikely in the TeV regime. Around one TeV, all the effects are substancially under control (e.g. below the ten percent level, assumed to be a rough threshold for the reliability of the approximation). In the CLIC region (3-5 TeV) the ten percent boundary is systematically crossed, and the effect in the MSSM can reach values varying from fifteen to twentyfive percent in the cross section, depending on $\tan\beta$ (the SM effect is smaller in this case; an opposite situation characterizes the two polarization asymmetries). For the specific purposes of a very high precision test, this would strongly motivate a (hard)

two loop calculation of the most relevant (in practice, the logarithms of "Sudakov-type") effects [14]. On the other hand, one could argue that possible neglected terms, e.g. constant ones, might somehow reduce the size of the effect. Our personal feeling, motivated by the previous experience for light fermion production [1], [2], is that in the SM case these extra terms can reduce the effect, but not drastically (i.e. at the few percent reduction level). In the MSSM case, this feeling remains to be investigated in some more detail, although it is confirmed by a partial previous analysis performed in Ref. [3]. Certainly, if one moves to the 10 TeV region, where the leading logarithms should provide a rather reliable approximation, the forty percent relative effect in the top cross section shown in Fig.3 appears hardly compatible with a one-loop truncation of the electroweak perturbative expansion.

4) The strong dependence on $\tan\beta$ of the leading asymptotic terms appears to be a special characteristic of the top production in the TeV regime. This is a consequence of the "massive" linear logarithms of "Sudakov-type", proportional to m_t^2 (and also in the SUSY case, to m_b^2) and generated by the final top vertex. To visualize the numerical dependence, we have plotted in the final Fig.11-13 the variation of the leading effects with $\tan\beta$, at the CLIC reference point" $\sqrt{q^2} = 3\ TeV$. As one sees, the numerical dependence has some features that appear to us potentially interesting. Assuming a typical "visibility parameter" of one percent, that corresponds to an integrated luminosity of about $10^3\ fb^{-1}$, we notice that:

a) The effect should be largely visible (e.g. it is around the 10 percent level when $tan\beta$ varies from 1 to 10) in the case of the cross section σ_t; it remains less strongly but still potentially visible in the polarized asymmetries A_t^{LR} and A_t (around the few percent level varying $tan\beta$ from 1 to 10); it is irrelevant (10 times smaller) in $A_{t,FB}$.

b) It varies from -14 to -5 and from -5 to -9 percent in σ_t and from $+4$ to $+1$ and from $+1$ to -1 percent in the polarized asymmetries A_t^{LR} and A_t when $\tan\beta$ varies from 1 to 10 and from 10 to 40. Therefore, it is in principle sensitive to the large $tan\beta$ region.

These features, if retained by a more complete approximation e.g. that includes possible constant terms, would make top production in the CLIC regime a promising and, in a certain sense, unique "$\tan\beta$ detector". A dedicated analysis with this specific purpose is already being actively performed [9].

Acknowledgments: This work has been partially supported by the European Community grant HPRN-CT-2000-00149.

APPENDIX A: ASYMPTOTIC LOGARITHMIC CONTRIBUTIONS IN THE MSSM

1. SM contributions

In order to allow an easy comparison of the above SUSY contributions with the SM ones we now recall, in the next three subsections, the results obtained in [1], [2] for the same four gauge invariant functions.

(a) Universal SM contributions

$$\tilde{\Delta}^{(RG)}_{\alpha}(q^2,\theta) \to \frac{\alpha(\mu^2)}{12\pi}\left(\frac{32}{3}N-21\right)\ln(\frac{q^2}{\mu^2}) \tag{A1}$$

$$R^{(RG)}(q^2,\theta) \to -\frac{\alpha(\mu^2)}{4\pi s^2_W c^2_W}\left(\frac{20-40c^2_W+32c^4_W}{9}N+\frac{1-2c^2_W-42c^4_W}{6}\right)\ln(\frac{q^2}{\mu^2}) \tag{A2}$$

$$V^{(RG)}_{\gamma Z}(q^2,\theta) = V^{(RG)}_{Z\gamma}(q^2,\theta) \to \frac{\alpha(\mu^2)}{3\pi s_W c_W}\left(\frac{10-16c^2_W}{6}N+\frac{1+42c^2_W}{8}\right)\ln(\frac{q^2}{\mu^2}) \tag{A3}$$

(b) $m_{t,b}$-independent terms in SM non-universal contributions to $l^+l^- \to t\bar{t}$ (same as in $u\bar{u}$, $c\bar{c}$)

$$\begin{aligned}\tilde{\Delta}^{(S)}_{\alpha,lf}(q^2,\theta) \to\ & \frac{5\alpha}{4\pi}\ln\frac{q^2}{M^2_W}+\frac{\alpha}{12\pi}\ln^2\frac{q^2}{M^2_W}+\frac{\alpha(2-v^2_l-v^2_t)}{64\pi s^2_W c^2_W}\left(3\ln\frac{q^2}{M^2_Z}-\ln^2\frac{q^2}{M^2_Z}\right)\\ &-\frac{\alpha}{2\pi}\left(\ln^2\frac{q^2}{M^2_W}+2\ln\frac{q^2}{M^2_W}\ln\frac{1+cos\theta}{2}\right)\\ &-\frac{\alpha}{256\pi Q_f s^4_W c^4_W}(1-v^2_l)(1-v^2_t)\ln\frac{q^2}{M^2_Z}\ln\frac{1+cos\theta}{1-cos\theta}\end{aligned} \tag{A4}$$

$$\begin{aligned}R^{(S)}_{lf}(q^2,\theta) \to\ & -\frac{3\alpha}{4\pi s^2_W}\left(1-\frac{5s^2_W}{3}\right)\ln\frac{q^2}{M^2_W}-\frac{\alpha}{4\pi s^2_W}\left(1-\frac{s^2_W}{3}\right)\ln^2\frac{q^2}{M^2_W}\\ &-\frac{\alpha(2+3v^2_l+3v^2_t)}{64\pi s^2_W c^2_W}\left(3\ln\frac{q^2}{M^2_Z}-\ln^2\frac{q^2}{M^2_Z}\right)+\frac{\alpha c^2_W}{2\pi s^2_W}\left(\ln^2\frac{q^2}{M^2_W}+2\ln\frac{q^2}{M^2_W}\ln\frac{1+cos\theta}{2}\right)\\ &+\frac{\alpha}{4\pi s^2_W c^2_W}v_l v_t\ln\frac{q^2}{M^2_Z}\ln\frac{1+cos\theta}{1-cos\theta}\end{aligned} \tag{A5}$$

$$V^{(S)}_{\gamma Z,lf}(q^2,\theta) \to \frac{\alpha}{8\pi c_W s_W}\left[(3-10c^2_W)\ln\frac{q^2}{M^2_W}-\left(1+\frac{2}{3}c^2_W\right)\ln^2\frac{q^2}{M^2_W}\right]$$

$$-\left[\frac{\alpha v_l(1-v_l^2)}{128\pi s_W^3 c_W^3}+\frac{\alpha|Q_t|v_t}{8\pi s_W c_W}\right]\left(3\ln\frac{q^2}{M_Z^2}-\ln^2\frac{q^2}{M_Z^2}\right)$$
$$+\frac{\alpha c_W}{2\pi s_W}\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1+cos\theta}{2}\right)$$
$$+\frac{\alpha}{32\pi s_W^3 c_W^3}v_t(1-v_l^2)\ln\frac{q^2}{M_Z^2}\ln\frac{1+cos\theta}{1-cos\theta} \quad \text{(A6)}$$

$$V^{(S)}_{Z\gamma,lf}(q^2,\theta)\rightarrow\frac{\alpha}{8\pi cs}\left[(10s_W^2-9)\ln\frac{q^2}{M_W^2}-\left(1-\frac{2}{3}s_W^2\right)\ln^2\frac{q^2}{M_W^2}\right]$$
$$-\left[\frac{\alpha v_t(1-v_t^2)}{128\pi|Q_t|s_W^3 c_W^3}+\frac{\alpha v_l}{8\pi s_W c_W}\right]\left(3\ln\frac{q^2}{M_Z^2}-\ln^2\frac{q^2}{M_Z^2}\right)$$
$$+\frac{\alpha c_W}{2\pi s_W}\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1+cos\theta}{2}\right)$$
$$+\frac{\alpha}{32\pi Q_t s_W^3 c_W^3}v_l(1-v_t^2)\ln\frac{q^2}{M_Z^2}\ln\frac{1+cos\theta}{1-cos\theta} \quad \text{(A7)}$$

where $v_l = 1-4\,s_W^2,\;\; v_t = 1-4\,|Q_t|\,s_W^2$.

In each of the above equations, we have successively added the contributions coming from triangles containing one or two W, from triangles containing one Z, from WW box and finally from ZZ box.

(c) $m_{t,b}$-dependent terms in non-universal SM contributions to $l^+l^-\rightarrow t\bar{t}$

$$\tilde{\Delta}_{\alpha,lt}(q^2)\rightarrow\tilde{\Delta}_{\alpha,lc}(q^2)-\frac{\alpha}{24\pi s_W^2}\ln q^2\left[(3-2s_W^2)\frac{m_t^2}{M_W^2}+2s_W^2\frac{m_b^2}{M_W^2}\right] \quad \text{(A8)}$$

$$R_{lt}(q^2)\rightarrow R_{lc}(q^2)+\frac{\alpha}{16\pi s_W^2}\ln q^2\left[\left(1+\frac{4s_W^2}{3}\right)\frac{m_t^2}{M_W^2}+\left(1-\frac{4s_W^2}{3}\right)\frac{m_b^2}{M_W^2}\right] \quad \text{(A9)}$$

$$V_{\gamma Z,lt}(q^2)\rightarrow V_{\gamma Z,lc}(q^2)-\frac{\alpha c_W}{12\pi s_W}\ln q^2\left(\frac{m_t^2}{M_W^2}-\frac{m_b^2}{M_W^2}\right) \quad \text{(A10)}$$

$$V_{Z\gamma,lt}(q^2)\rightarrow V_{Z\gamma,lc}(q^2)-\frac{\alpha}{16\pi s_W c_W}\ln q^2\left(1-\frac{4s_W^2}{3}\right)\left(\frac{m_t^2}{M_W^2}-\frac{m_b^2}{M_W^2}\right) \quad \text{(A11)}$$

2. Additional SUSY contributions

(a)Universal (γ, Z-self-energy) SUSY contributions

They arise from the bubbles (and associated tadpole diagrams) involving internal L- and R- sleptons and squarks, charginos, neutralinos, as well as the charged and neutral Higgses and Goldstones (subtracting the standard Higgs contribution):

$$\tilde{\Delta}_{\alpha}^{Univ}(q^2) \to \frac{\alpha}{4\pi}\left(3+\frac{16N}{9}\right)\ \ln q^2 \tag{A12}$$

$$R^{Univ}(q^2) \to -\frac{\alpha}{4\pi s_W^2 c_W^2}\left[\frac{13-26s_W^2+18s_W^4}{6}+(3-6s_W^2+8s_W^4)\frac{2N}{9}\right]\ \ln q^2 \tag{A13}$$

$$V_{\gamma Z}^{Univ}(q^2) = V_{Z\gamma}^{Univ}(q^2) \to -\frac{\alpha}{4\pi s_W c_W}\left[\frac{13-18s_W^2}{6}+(3-8s_W^2)\frac{2N}{9}\right]\ \ln q^2 \tag{A14}$$

where N is the number of slepton and squark families. These terms contribute to the RG effects.

We then consider the non-universal SUSY contributions. These are the contributions coming from triangle diagrams connected either to the initial l^+l^- or to the final $t\bar{t}$ lines, and containing SUSY partners, sfermions $\tilde{f}$, charginos or neutralinos χ_i, or SUSY Higgses (see Fig.2); external fermion self-energy diagrams are added making the total contribution finite. These non universal terms consist in $m_{t,b}$-independent terms and in $m_{t,b}$-dependent terms (quadratic m_t^2 and m_b^2 terms) given below:

(b) $m_{t,b}$-independent terms in SUSY contributions to $l^+l^- \to t\bar{t}$ (same as for $u\bar{u},\ c\bar{c}$)

$$\tilde{\Delta}_{\alpha,lt}(q^2) \to \frac{\alpha}{\pi}\ \ln q^2\ \frac{-71+82s_W^2}{72c_W^2} \tag{A15}$$

$$R_{lt}(q^2) \to \frac{\alpha}{\pi}\ \ln q^2\ \frac{27-67s_W^2+82s_W^4}{72s_W^2c_W^2} \tag{A16}$$

$$V_{\gamma Z,lt}(q^2) \to \frac{\alpha}{\pi}\ \ln q^2\ \frac{63-200s_W^2+164s_W^4}{144s_Wc_W^3} \tag{A17}$$

$$V_{Z\gamma,lt}(q^2) \to \frac{\alpha}{\pi}\ \ln q^2\ \frac{81-240s_W^2+164s_W^4}{144s_Wc_W^3} \tag{A18}$$

(c) $m_{t,b}$-dependent terms in SUSY contributions to $l^+l^- \to t\bar{t}$

$$\tilde{\Delta}_{\alpha,lt}(q^2) \to \tilde{\Delta}_{\alpha,lc}(q^2) - \frac{\alpha}{24\pi s_W^2}\ln q^2 \left[(3-2s_W^2)(1+2\cot^2\beta)\frac{m_t^2}{M_W^2} + 2s_W^2\frac{m_b^2}{M_W^2}(1+2\tan^2\beta)\right] \quad (A19)$$

$$R_{lt}(q^2) \to R_{lc}(q^2) + \frac{\alpha}{16\pi s_W^2}\ln q^2 \left[\left(1+\frac{4s_W^2}{3}\right)(1+2\cot^2\beta)\frac{m_t^2}{M_W^2} + \left(1-\frac{4s_W^2}{3}\right)\frac{m_b^2}{M_W^2}(1+2\tan^2\beta)\right] \quad (A20)$$

$$V_{\gamma Z,lt}(q^2) \to V_{\gamma Z,lc}(q^2) - \frac{\alpha c_W}{12\pi s_W}\ln q^2 \left(\frac{m_t^2}{M_W^2}(1+2\cot^2\beta) - \frac{m_b^2}{M_W^2}(1+2\tan^2\beta)\right) \quad (A21)$$

$$V_{Z\gamma,lt}(q^2) \to V_{Z\gamma,lc}(q^2) - \frac{\alpha}{16\pi s_W c_W}\ln q^2 \left(1-\frac{4s_W^2}{3}\right)\left(\frac{m_t^2}{M_W^2}(1+2\cot^2\beta) - \frac{m_b^2}{M_W^2}(1+2\tan^2\beta)\right) \quad (A22)$$

3. Non-universal massive MSSM contribution

Finally we find interesting to sum up all the massive m_t^2 and m_b^2 terms appearing in the MSSM (SM and SUSY non-universal massive contributions to $l^+l^- \to t\bar{t}$). We remark that the net effect as compared to the SM result is a factor $2(1+\cot^2\beta)$ for the m_t^2 term and a factor $2(1+\tan^2\beta)$ for the m_b^2 one, a rule similar to the one observed in the $b\bar{b}$ case [3]:

$$\tilde{\Delta}_{\alpha,lt}(q^2) \to \tilde{\Delta}_{\alpha,lc}(q^2) - \frac{\alpha}{12\pi s_W^2}\ln q^2 \left[(3-2s_W^2)(1+\cot^2\beta)\frac{m_t^2}{M_W^2} + 2s_W^2\frac{m_b^2}{M_W^2}(1+\tan^2\beta)\right] \quad (A23)$$

$$R_{lt}(q^2) \to R_{lc}(q^2) + \frac{\alpha}{8\pi s_W^2}\ln q^2 \left[\left(1+\frac{4s_W^2}{3}\right)(1+\cot^2\beta)\frac{m_t^2}{M_W^2} + \left(1-\frac{4s_W^2}{3}\right)\frac{m_b^2}{M_W^2}(1+\tan^2\beta)\right] \quad (A24)$$

$$V_{\gamma Z,lt}(q^2) \to V_{\gamma Z,lc}(q^2) - \frac{\alpha c_W}{6\pi s_W}\ln q^2 \left(\frac{m_t^2}{M_W^2}(1+\cot^2\beta) - \frac{m_b^2}{M_W^2}(1+\tan^2\beta)\right) \quad (A25)$$

$$V_{Z\gamma,lt}(q^2) \to V_{Z\gamma,lc}(q^2) - \frac{\alpha}{8\pi s_W c_W}\ln q^2 \left(1-\frac{4s_W^2}{3}\right)\left(\frac{m_t^2}{M_W^2}(1+\cot^2\beta) - \frac{m_b^2}{M_W^2}(1+\tan^2\beta)\right) \quad (A26)$$

APPENDIX B: HELICITY AMPLITUDES AND TOP SPIN DENSITY MATRIX

A generic invariant amplitude

$$A=\frac{e^2}{q^2}\bar{u}(t)[\gamma^\mu g^\gamma_{Vt}+\frac{d^\gamma}{m_t}(p-p')_\mu]v(\bar{t})\ .\ \bar{v}(l)\gamma_\mu g^\gamma_{Vl}u(l)+$$
$$\frac{e^2}{4s^2_W c^2_W(q^2-m^2_Z)}\bar{u}(t)[\gamma^\mu(g^Z_{Vt}-g^Z_{At}\gamma^5)+\frac{d^Z}{m_t}(p-p')^\mu]v(\bar{t})\ .\ \bar{v}(l)\gamma_\mu(g^Z_{Vl}-g^Z_{Al}\gamma^5)u(l) \qquad \text{(B1)}$$

leads to the helicity amplitude

$$F(\lambda_l,\lambda_t,\lambda_{\bar{t}})=(2\lambda_l)e^2\sqrt{q^2}.$$
$$.\{\ \frac{g^\gamma_{Vl}}{q^2}[\ g^\gamma_{Vt}(2m_t sin\theta\delta_{\lambda_t,\lambda_{\bar{t}}}-\sqrt{q^2}(\lambda_t-\lambda_{\bar{t}})cos\theta-2\lambda_l\sqrt{q^2}\delta_{\lambda_t,-\lambda_{\bar{t}}})-\frac{d^\gamma}{m_t}\beta^2_t q^2 sin\theta\delta_{\lambda_t,\lambda_{\bar{t}}}]$$
$$+\frac{g^Z_{Vl}-2\lambda g^Z_{Al}}{4s^2_W c^2_W(q^2-m^2_Z)}[g^Z_{Vt}(2m_t sin\theta\delta_{\lambda_t,\lambda_{\bar{t}}}-\sqrt{q^2}(\lambda_t-\lambda_{\bar{t}})cos\theta-2\lambda\sqrt{q^2}\delta_{\lambda_t,-\lambda_{\bar{t}}})$$
$$+g^Z_{At}\beta_t\sqrt{q^2}(cos\theta\delta_{\lambda_t,-\lambda_{\bar{t}}}+2\lambda_l(\lambda_t-\lambda_{\bar{t}})-\frac{d^Z}{m_t}\beta^2_t q^2 sin\theta\delta_{\lambda_t,\lambda_{\bar{t}}}]\} \qquad \text{(B2)}$$

where $\lambda_l\equiv\lambda_{l^-}\equiv-\lambda_{l^+}=\pm\frac{1}{2}$, $\lambda_t=\pm\frac{1}{2}$, $\lambda_{\bar{t}}=\pm\frac{1}{2}$ and the normalization is such that the differential cross section is given by

$$\frac{d\sigma}{dcos\theta}=\frac{\beta_t N_t}{64\pi q^2}\{(1-PP')[\rho^U(+,+)+\rho^U(-,-)]+(P-P')[\rho^{LR}(+,+)+\rho^{LR}(-,-)]\} \quad \text{(B3)}$$

with $\beta_t=\sqrt{1-\frac{4m^2_t}{q^2}}$. N_t is the colour factor 3, times the QCD correction factor. The top quark spin density matrix is defined as

$$\rho^U(\lambda_t,\lambda'_t)\equiv\frac{1}{2}\rho^{L+R}(\lambda_t,\lambda'_t)=\frac{1}{2}\sum_{\lambda_{\bar{t}}}[F(\lambda_l=-\frac{1}{2},\lambda_t,\lambda_{\bar{t}})F^*(\lambda_l=-\frac{1}{2},\lambda'_t,\lambda_{\bar{t}})$$
$$+F(\lambda_l=+\frac{1}{2},\lambda_t,\lambda_{\bar{t}})F^*(\lambda_l=+\frac{1}{2},\lambda'_t,\lambda_{\bar{t}})] \qquad \text{(B4)}$$

for unpolarized $l^\pm$ beams, and

$$\rho^{LR}(\lambda_t,\lambda'_t)\equiv\frac{1}{2}\rho^{L-R}(\lambda_t,\lambda'_t)=\frac{1}{2}\sum_{\lambda_{\bar{t}}}[F(\lambda_l=-\frac{1}{2},\lambda_t,\lambda_{\bar{t}})F^*(\lambda_l=-\frac{1}{2},\lambda'_t,\lambda_{\bar{t}})$$
$$-F(\lambda_l=+\frac{1}{2},\lambda_t,\lambda_{\bar{t}})F^*(\lambda_l=+\frac{1}{2},\lambda'_t,\lambda_{\bar{t}})] \qquad \text{(B5)}$$

for longitudinally polarized l^- and l^+ beams with degrees P and P', respectively.
From eq.(B3) one constructs the usual four observables σ_t, $A_{FB,t}$, $A_{LR,t}$ and A_t. But from the density matrices eq.(B4, B5) one can also construct other observables related to the final top quark polarization.

At large q^2 the top quark polarization is only described by its helicity (the transverse polarization vanishes). So one defines the following observables:

(a) with unpolarized l^+l^- beams:
— the averaged top helicity

$$H_t = \frac{\int \ [\rho^U(+,+) - \rho^U(-,-)] \ dcos\theta}{\int \ [\rho^U(+,+) + \rho^U(-,-)] \ dcos\theta} \tag{B6}$$

— its forward-backward asymmetry

$$H_{t,FB} = \frac{\int_{F-B} \ [\rho^U(+,+) - \rho^U(-,-)] \ dcos\theta}{\int \ [\rho^U(+,+) + \rho^U(-,-)] \ dcos\theta} \tag{B7}$$

(b) and the same two quantities for the Left-Right l^- polarization asymmetry
— the averaged polarized top helicity

$$H_t^{LR} = \frac{\int \ [\rho^{LR}(+,+) - \rho^{LR}(-,-)] dcos\theta}{\int \ [\rho^U(+,+) + \rho^U(-,-)] \ dcos\theta} \tag{B8}$$

— and its forward-backward asymmetry

$$H_{t,FB}^{LR} = \frac{\int_{F-B} \ [\rho^{LR}(+,+) - \rho^{LR}(-,-)] \ dcos\theta}{\int \ [\rho^U(+,+) + \rho^U(-,-)] \ dcos\theta} \tag{B9}$$

These quantities can be measured through the decay of the top quark into $W + b$, see for example the discussion in ref. [12].

In the asymptotic regime, the Z-peak subtraction method described in Section II implies the following expressions for the effective photon and Z couplings to be used in eq.(B2):

$$g^\gamma_{Vl} \rightarrow Q_e \left[1 + \frac{1}{2}\tilde{\Delta}_{\alpha,lt}(q^2,\theta)\right] \tag{B10}$$

$$g^\gamma_{Vt} \rightarrow Q_t \left[1 + \frac{1}{2}\tilde{\Delta}_{\alpha,lt}(q^2,\theta)\right] \tag{B11}$$

$$\frac{|e|}{2s_W c_W} g^Z_{Vl} \rightarrow -\ \frac{\tilde{v}_l}{2}\gamma_l^{\frac{1}{2}} \left[1 - \frac{1}{2}\hat{R}_{lt}(q^2,\theta) - \frac{4 s_W c_W}{\tilde{v}_l} V_{lt}^{\gamma Z}\right] \tag{B12}$$

$$\frac{|e|}{2s_W c_W} g^Z_{Al} \rightarrow -\ \frac{1}{2}\gamma_l^{\frac{1}{2}} \left[1 - \frac{1}{2}\hat{R}_{lt}(q^2,\theta)\right] \tag{B13}$$

$$\frac{|e|}{2s_W c_W} g^Z_{Vt} \rightarrow \frac{\tilde{v}_c}{2}\gamma_c^{\frac{1}{2}} \left[1 - \frac{1}{2}\hat{R}_{lt}(q^2,\theta) - \frac{8 s_W c_W}{3\tilde{v}_c} \hat{V}_{lt}^{Z\gamma}\right] \tag{B14}$$

$$\frac{|e|}{2s_W c_W} g^Z_{At} \to \frac{1}{2}\gamma_c^{\frac{1}{2}} \left[1 - \frac{1}{2}\hat{R}_{lt}(q^2, \theta)\right] \tag{B15}$$

where $\tilde{v}_f = 1 - 4|Q_f|s^2_{W,f}$, $s^2_{W,f}$ being the effective angle measured at Z peak in the channel $l^+l^- \to f\bar{f}$, and

$$\gamma_f \equiv \frac{48\pi\Gamma(Z \to f\bar{f})}{N_{Zf} M_Z (1 + \tilde{v}_f^2)} \tag{B16}$$

N_{Zf} being the colour factor 3, times the QCD correction factor in the $f\bar{f}$ channel at Z-peak.

Complete results including non asymptotic contributions and new Lorentz structures will be given in the forthcoming paper [9].

REFERENCES

[1] M. Beccaria, P. Ciafaloni, D. Comelli, F.M. Renard and C. Verzegnassi, Phys. Rev. **D61**,073005(2000).

[2] M. Beccaria, P. Ciafaloni, D. Comelli, F.M. Renard and C. Verzegnassi, Phys.Rev. **D61**,011301(2000).

[3] M. Beccaria, F.M. Renard and C. Verzegnassi, hep-ph/0007224.

[4] V. V. Sudakov, Sov. Phys. JETP 3, 65 (1956); Landau-Lifshits: Relativistic Quantum Field theory IV tome, ed. MIR.

[5] " The CLIC study of a multi-TeV e^+e^- linear collider", CERN-PS-99-005-LP (1999).

[6] $\mu^+\mu^-$ Collider: A feasibility Study. BNL-52503, Fermilab-Conf-96/092, LBNL-38946(1996); D.B. Cline, Int. J. Mod. Phys. bf A13,183(1998); and references to earlier workshops therein. See also Prospective Study of Muon Storage Rings in Europe, B. Autin et al, CERN 99-02.

[7] F.M. Renard and C. Verzegnassi, Phys. Rev. **D52**, 1369 (1995), Phys. Rev. **D53**, 1290 (1996).

[8] F.M. Renard and C. Verzegnassi, Phys.Rev.**D55**,4370(1997); A. Blondel, F.M. Renard, L. Trentadue and C. Verzegnassi, Phys.Rev **D54**,5567(1996); R.S. Chivukula, F.M. Renard and C. Verzegnassi, Phys.Rev**D57**,2760(1998).;M. Beccaria, F. Renard, S. Spagnolo, C. Verzegnassi, Phys. Lett.**B475**,157(2000); Phys.Rev.**D62**,053003(2000).

[9] M. Beccaria, F.M. Renard and C. Verzegnassi, in preparation.

[10] G. Degrassi and A. Sirlin, Nucl. Phys. B **383**, 73 (1992); Phys. Rev. D **46**, 3104 (1992).

[11] A.Blondel, B.W.Lynn, F.M.Renard and C.Verzegnassi, Nucl.Phys.**B304**,438(1988).

[12] G.J. Gounaris, J. Layssac and F.M. Renard, Phys.Rev.**D55**,5786(1997).

[13] F.M. Renard, "Basics of electron positron collisions, ed. Frontières (1981).

[14] M. Hori, H. Kawamura and J. Kodaira, hep-ph/0007329; P. Ciafaloni and D. Comelli, *Phys. Lett.* **B476** (2000) 49; J. H. Kühn and A. A. Penin, hep-ph/9906545; J. H. Kühn, A. A. Penin and V. A. Smirnov, hep-ph/9912503; V. S. Fadin, L. N. Lipatov, A. D. Martin and M. Melles, *Phys. Rev.* **D61** (2000) 094002; M. Melles, hep-ph/0004056; W. Beenakker and A. Werthenbach, hep-ph/0005316;

FIGURES

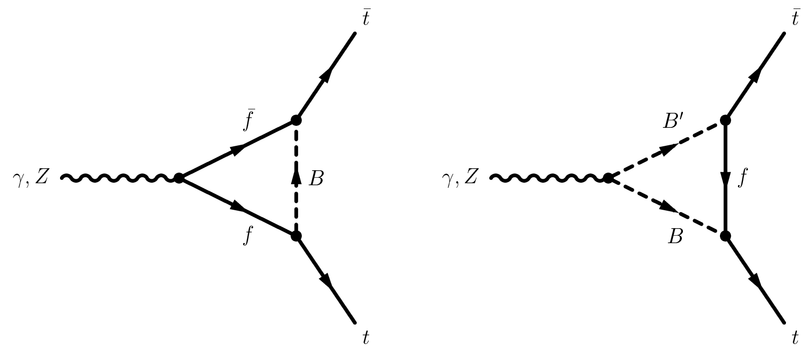

FIG. 1. Triangle SM diagrams contributing to the asymptotic logarithmic behaviour in the energy; f represent t or b quarks, B represent $W^{\pm}$, $\Phi^{\pm}$ or Z, G^0, H_{SM}.

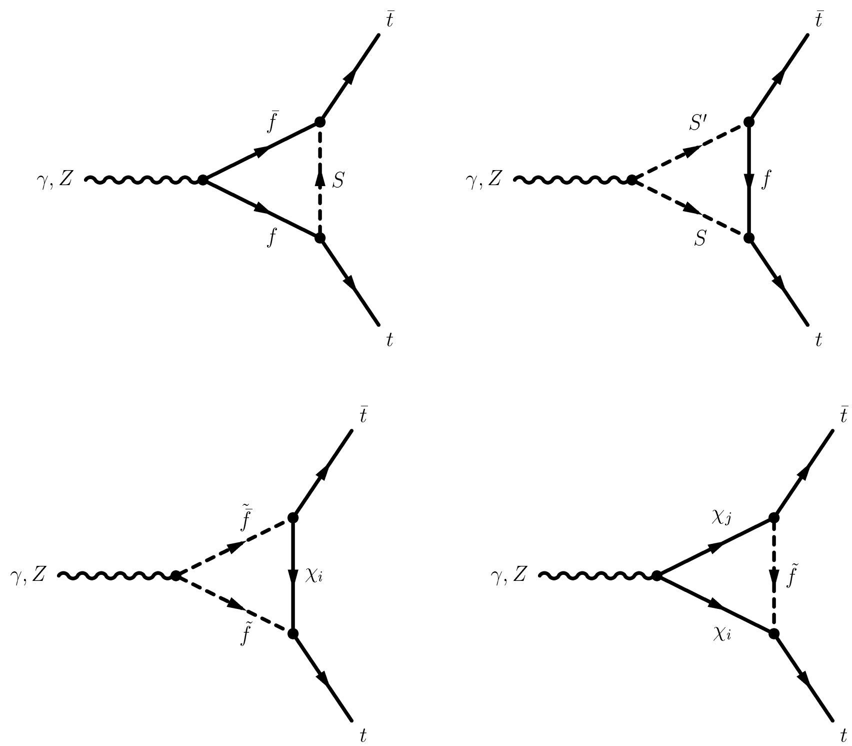

FIG. 2. Triangle diagrams with SUSY Higgs and with SUSY partners contributing to the asymptotic logarithmic behaviour in the energy; f represent t or b quarks; S represent charged or neutral Higgs bosons $H^{\pm}$, A^0, H^0, h^0 or Goldstone G^0; $\tilde{f}$ represent stop or sbottom states; χ represent charginos or neutralinos.

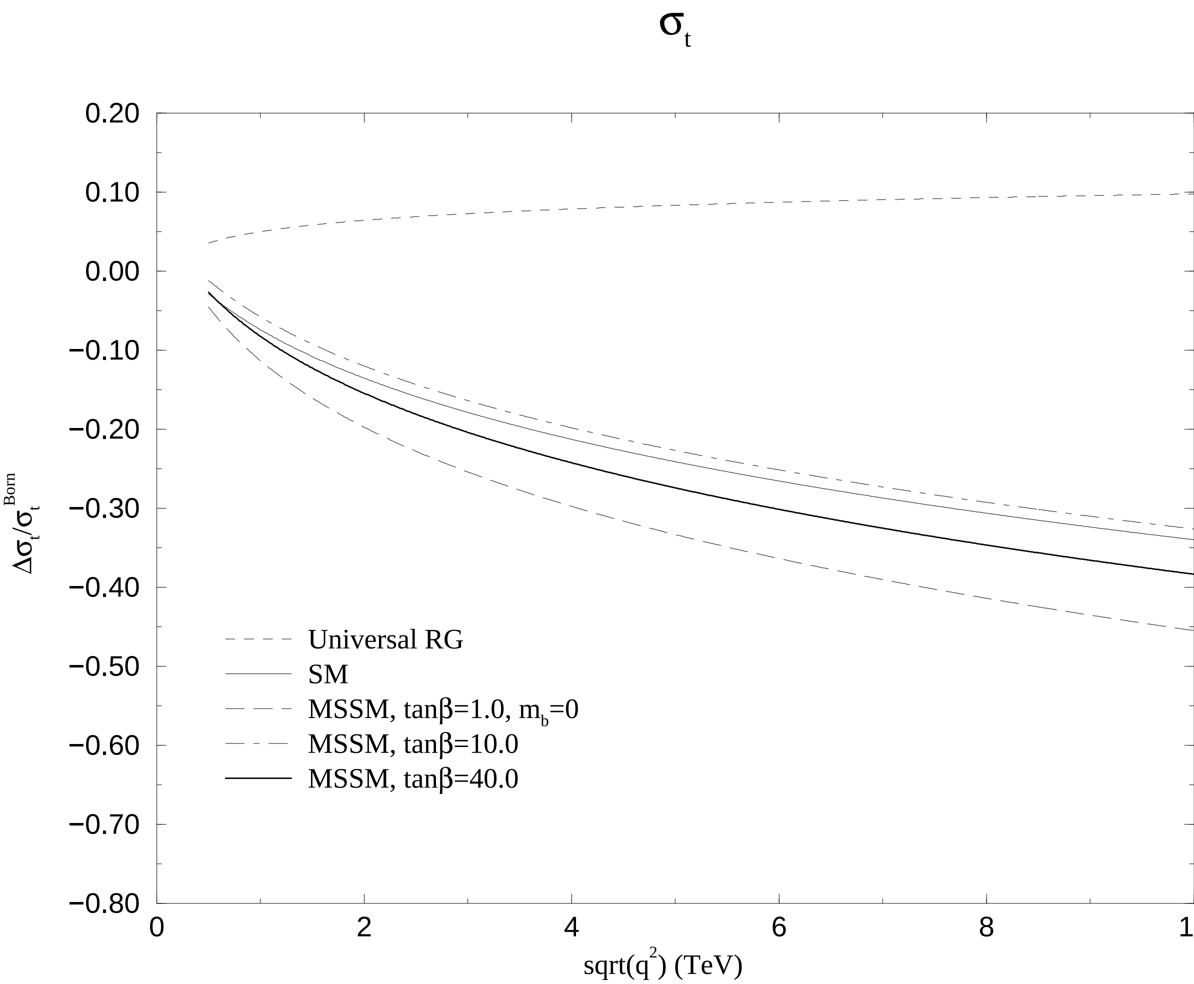

FIG. 3. Relative effects in σ_t due to the asymptotic logarithmic terms. The Born expression for large q^2 is 182 fb/(q^2/TeV^2).

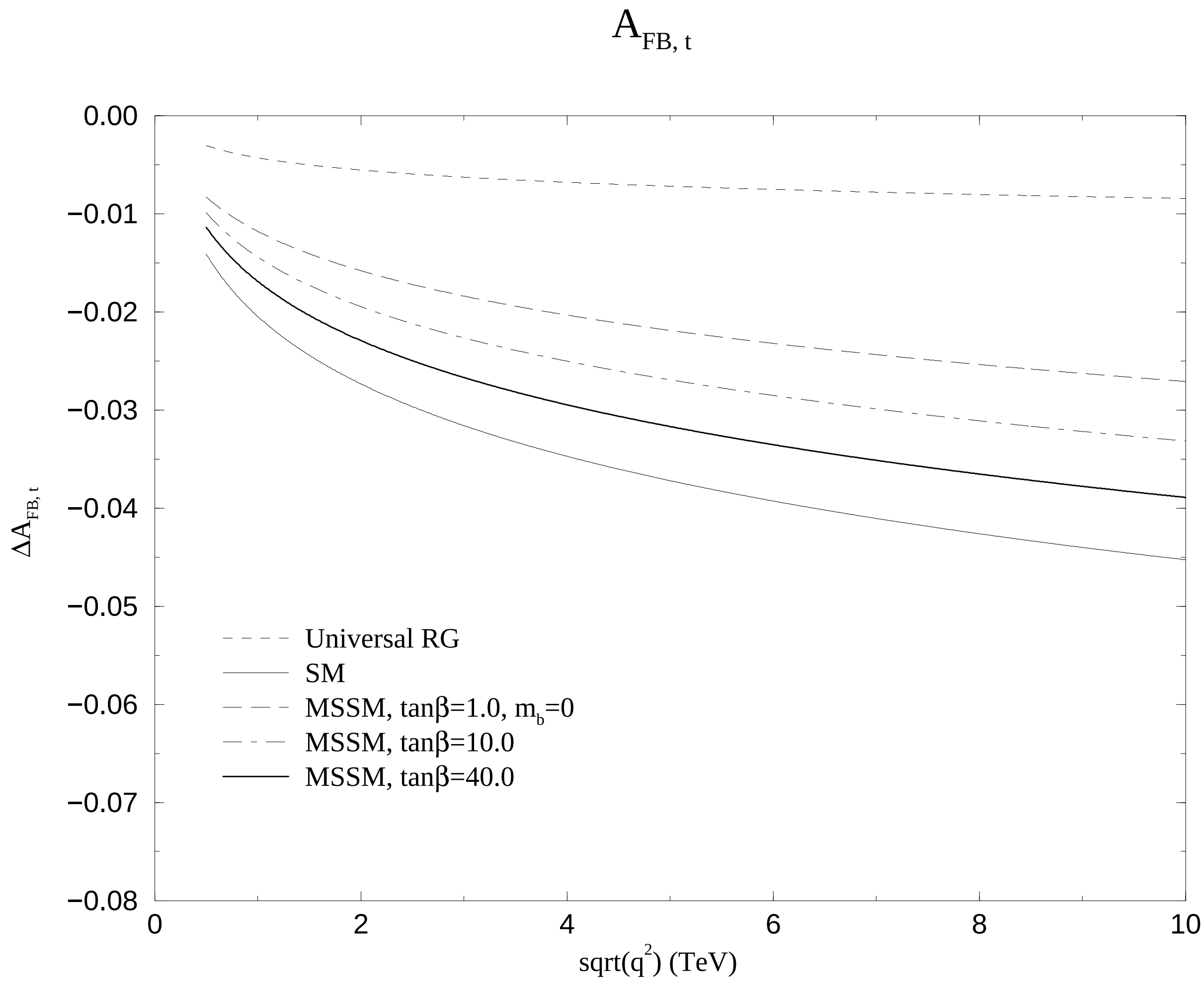

FIG. 4. Absolute effects in $A_{FB,t}$ due to the asymptotic logarithmic terms. The Born value for large q^2 is 0.607.

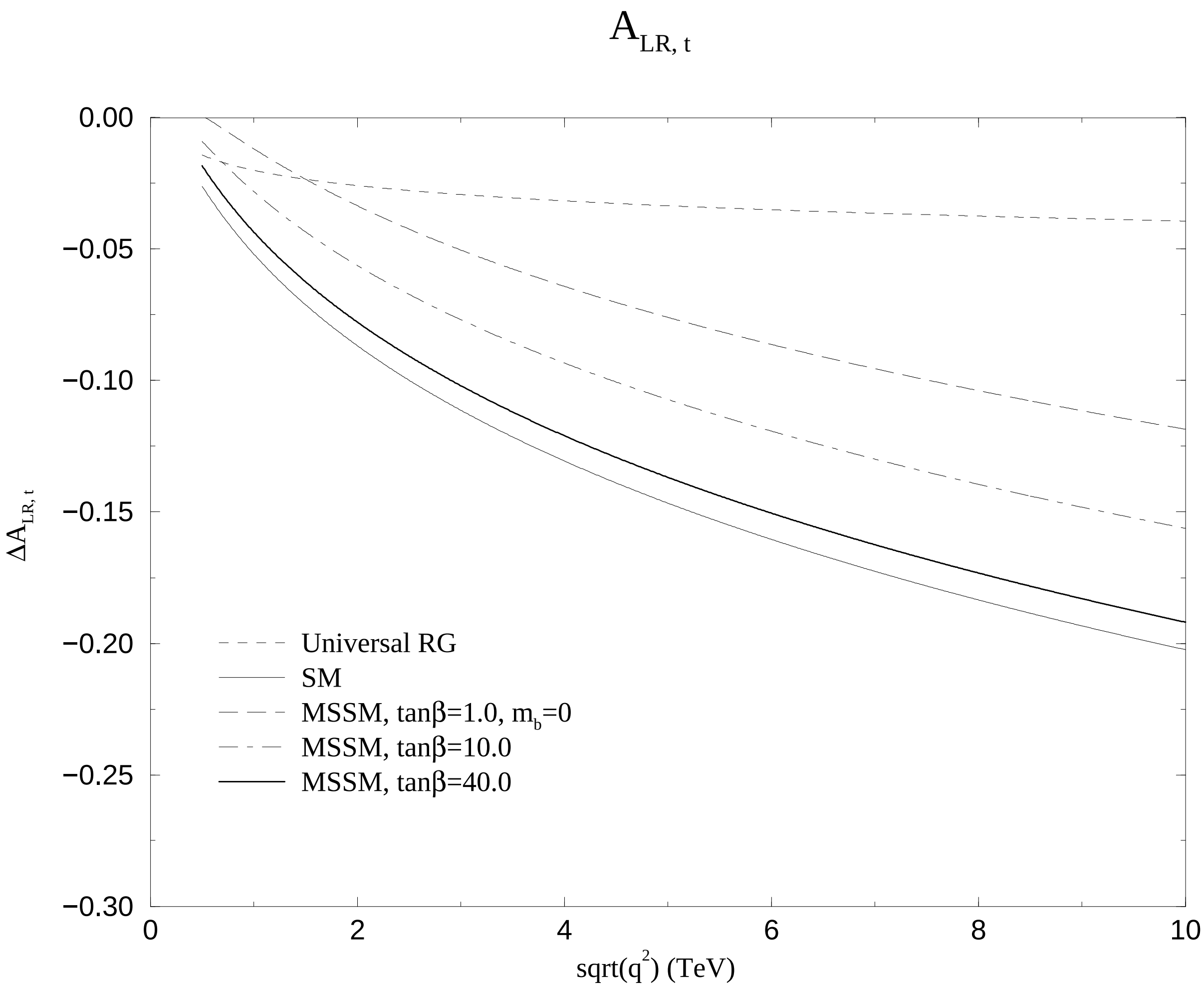

FIG. 5. Absolute effects in $A_{LR,t}$ due to the asymptotic logarithmic terms. The Born value for large q^2 is 0.336.

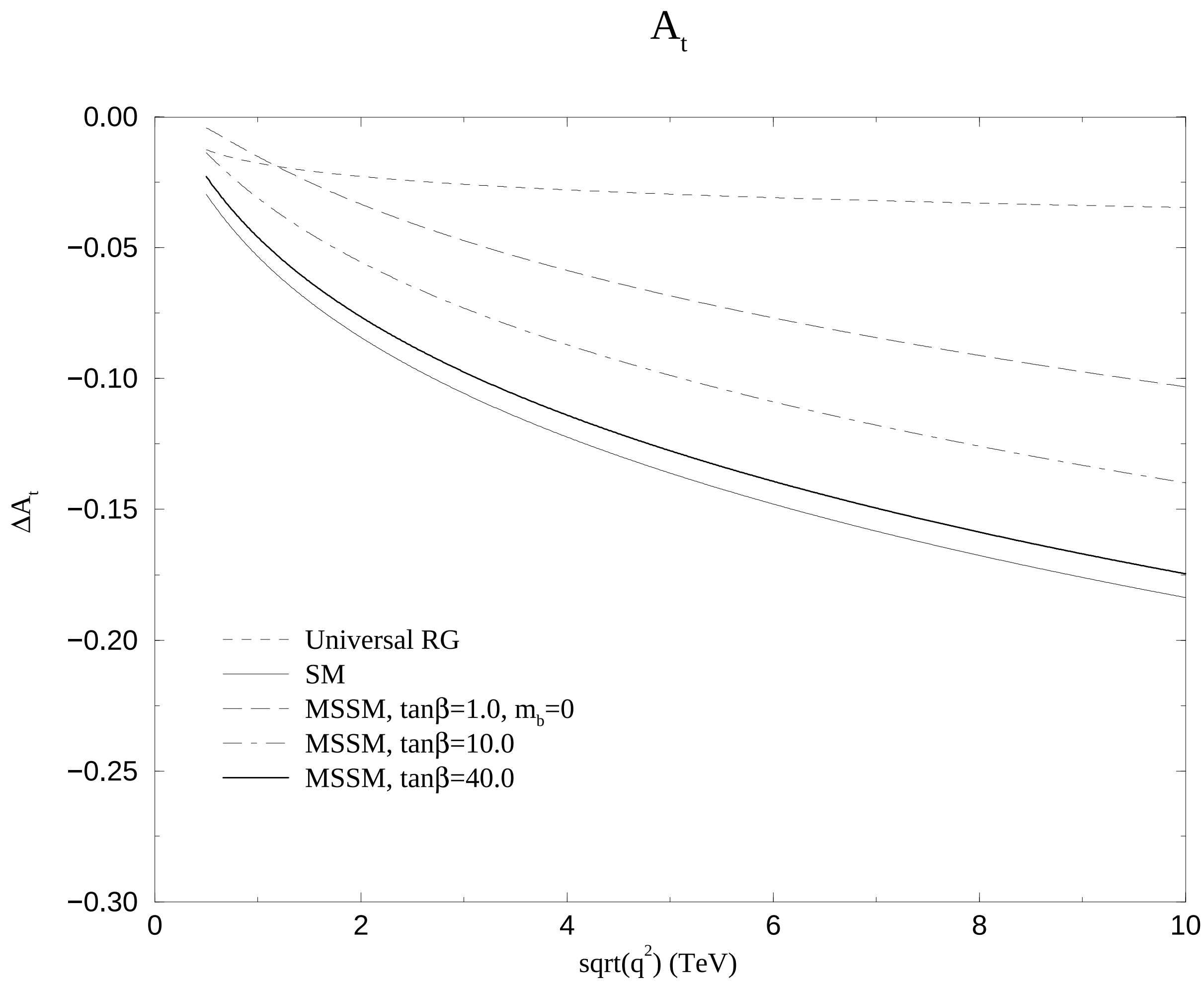

FIG. 6. Absolute effects in A_t due to the asymptotic logarithmic terms. The Born value for large q^2 is 0.164.

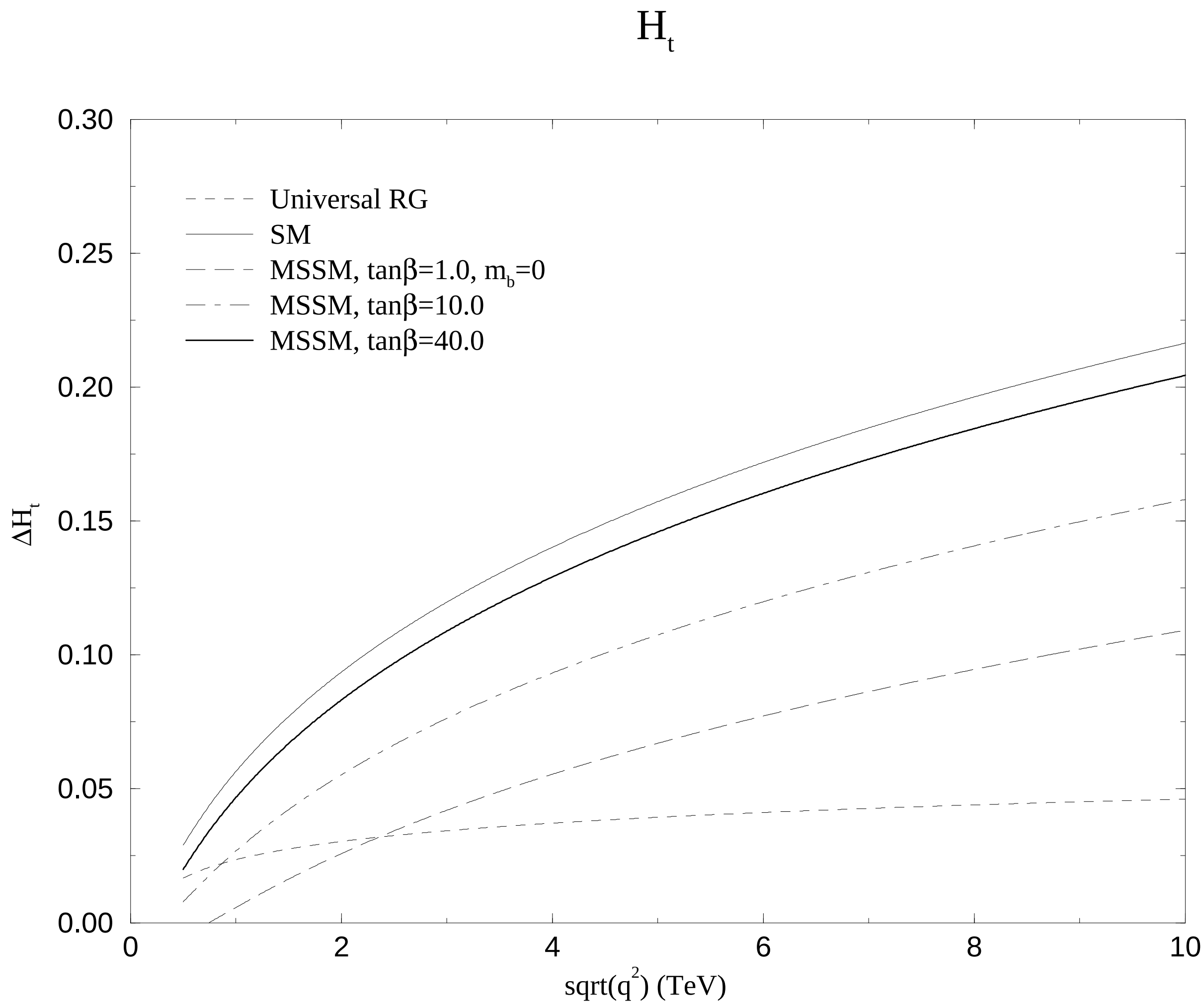

FIG. 7. Absolute effects in H_t due to the asymptotic logarithmic terms. The Born value for large q^2 is -0.219.

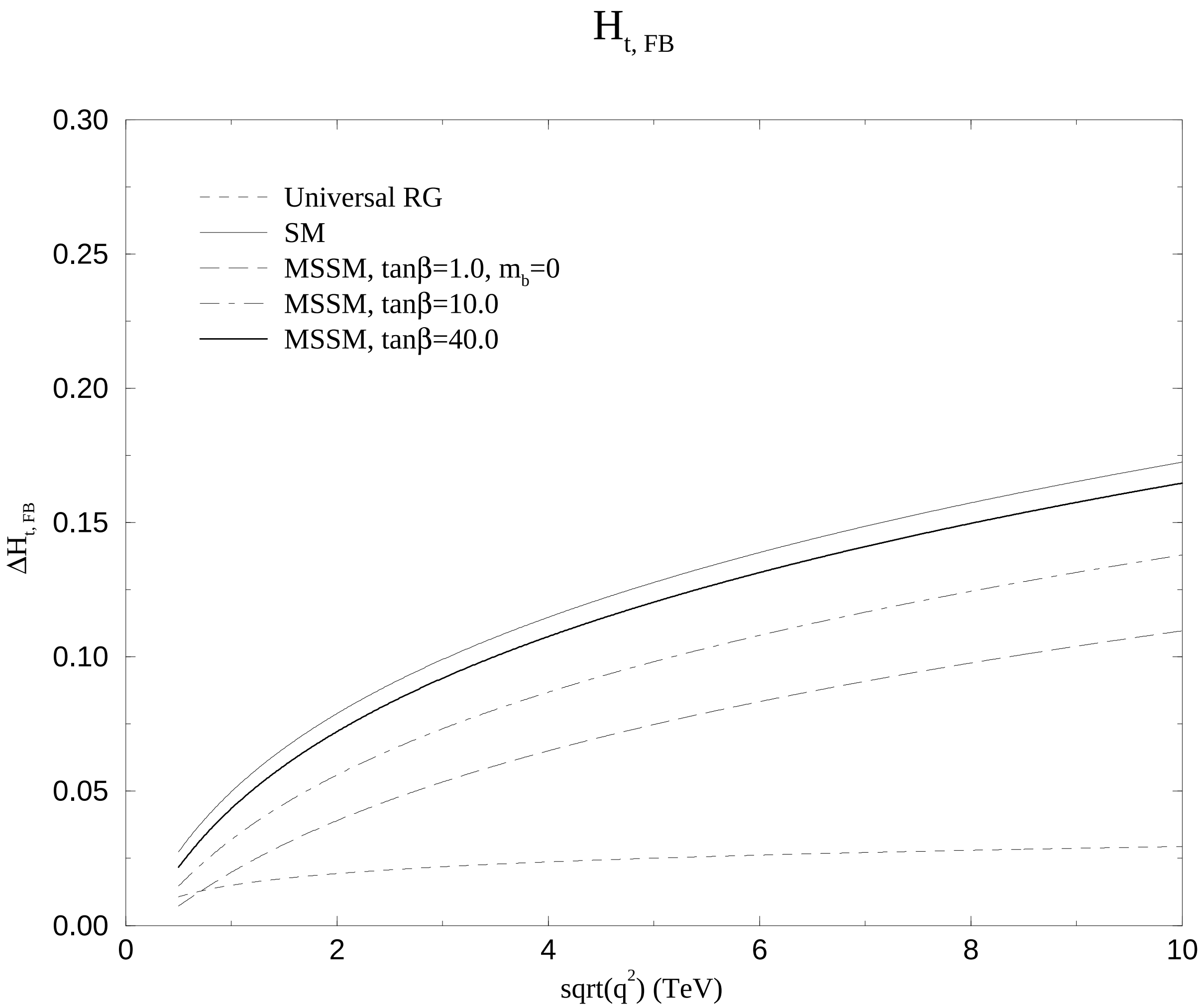

FIG. 8. Absolute effects in $H_{t,FB}$ due to the asymptotic logarithmic terms. The Born value for large q^2 is -0.252.

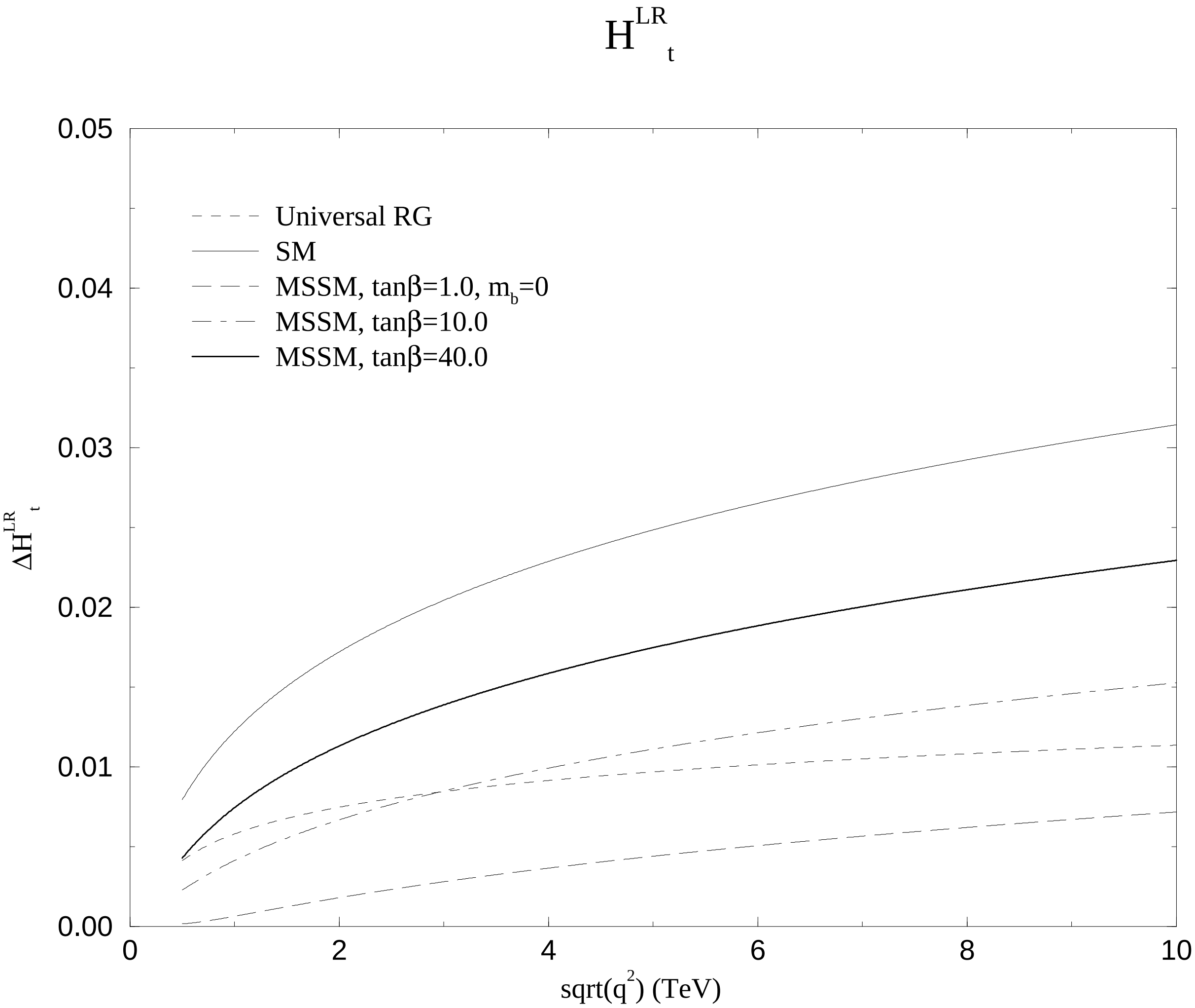

FIG. 9. Absolute effects in H_t^{LR} due to the asymptotic logarithmic terms. The Born value for large q^2 is -0.809.

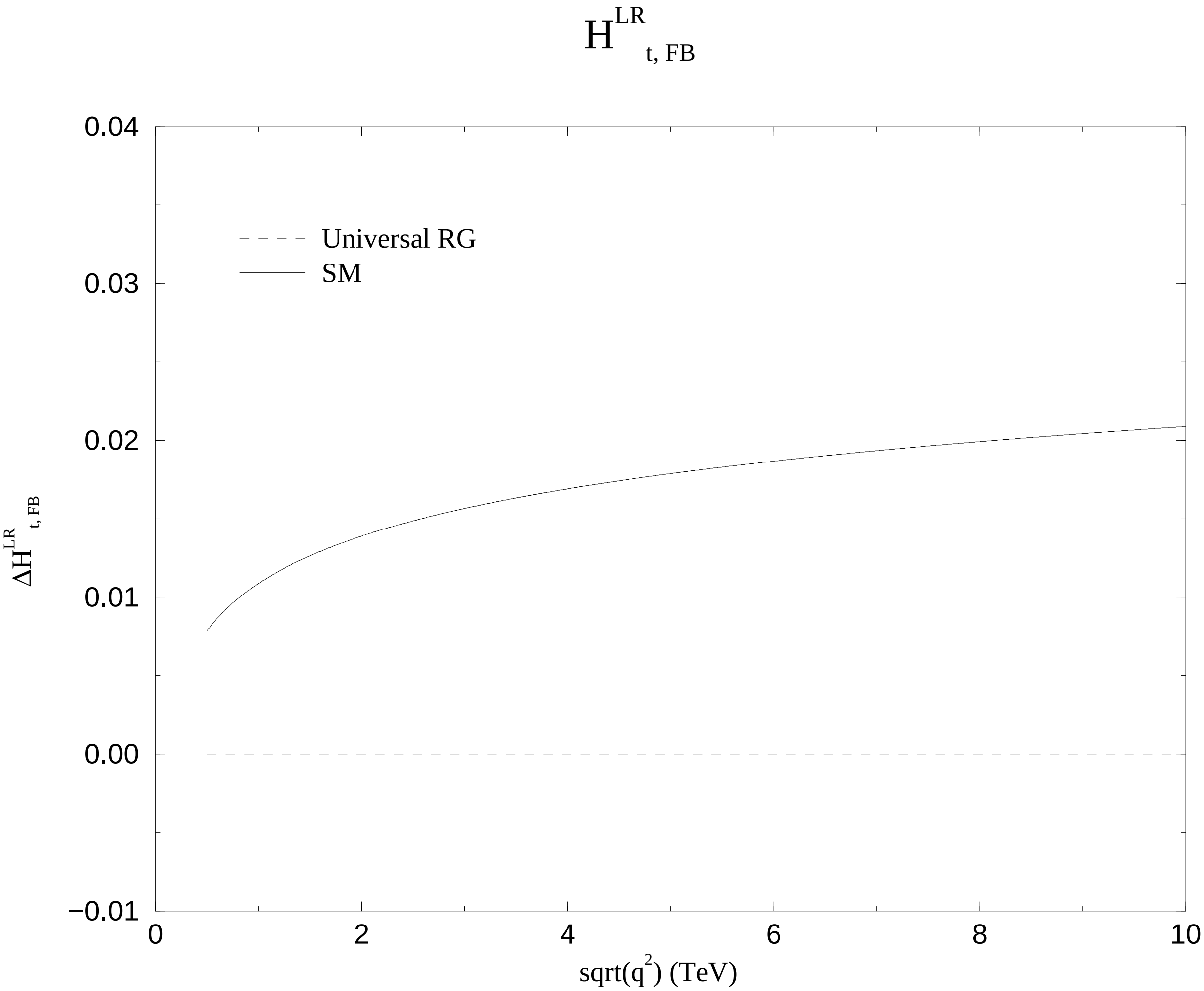

FIG. 10. Absolute effects in $H^{LR}_{t,FB}$ due to the asymptotic logarithmic terms (only SM box terms contribute). The Born value for large q^2 is -0.750.

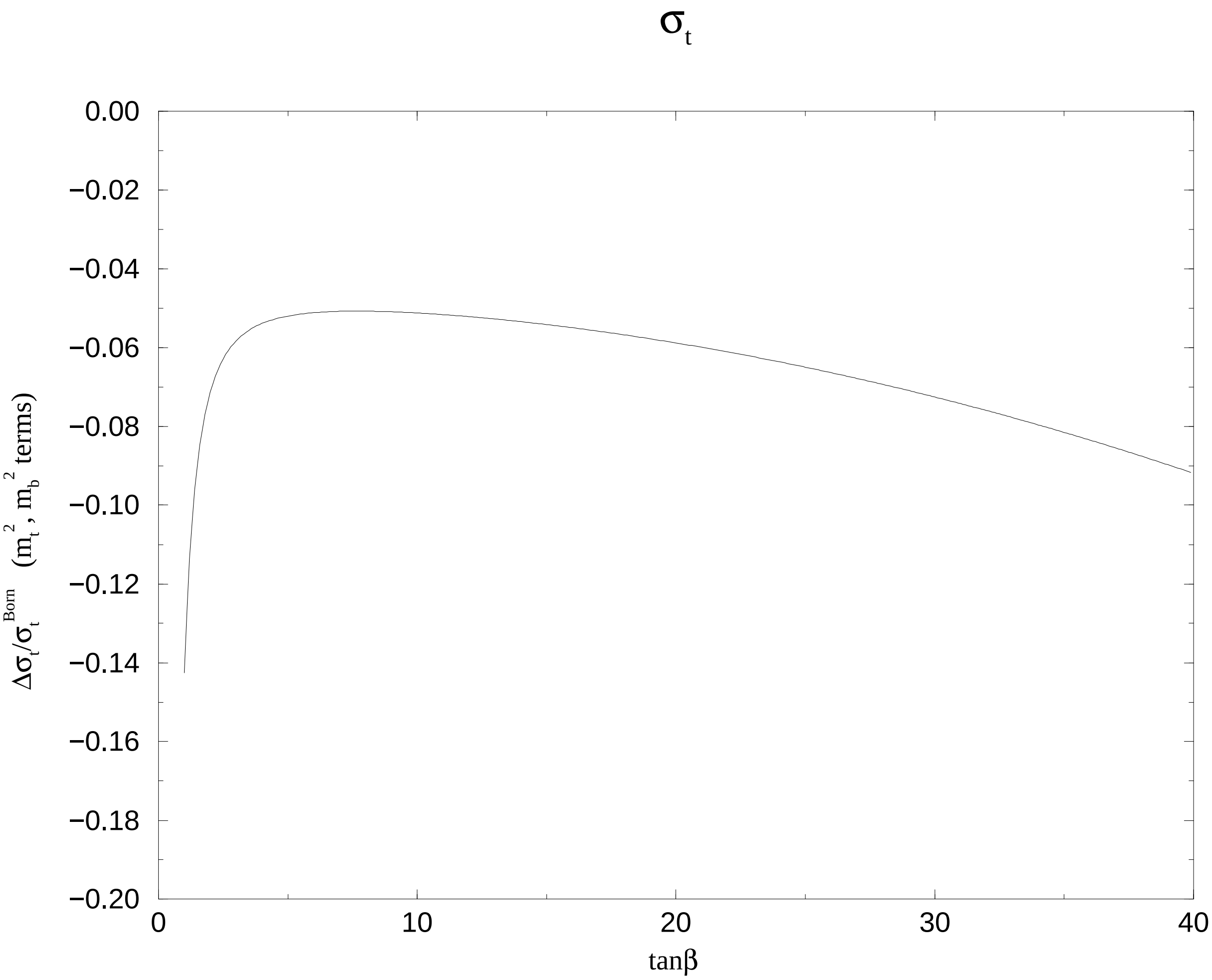

FIG. 11. Relative effects in σ_t due to the asymptotic m_t^2 and m_b^2 logarithmic terms versus $\tan\beta$.

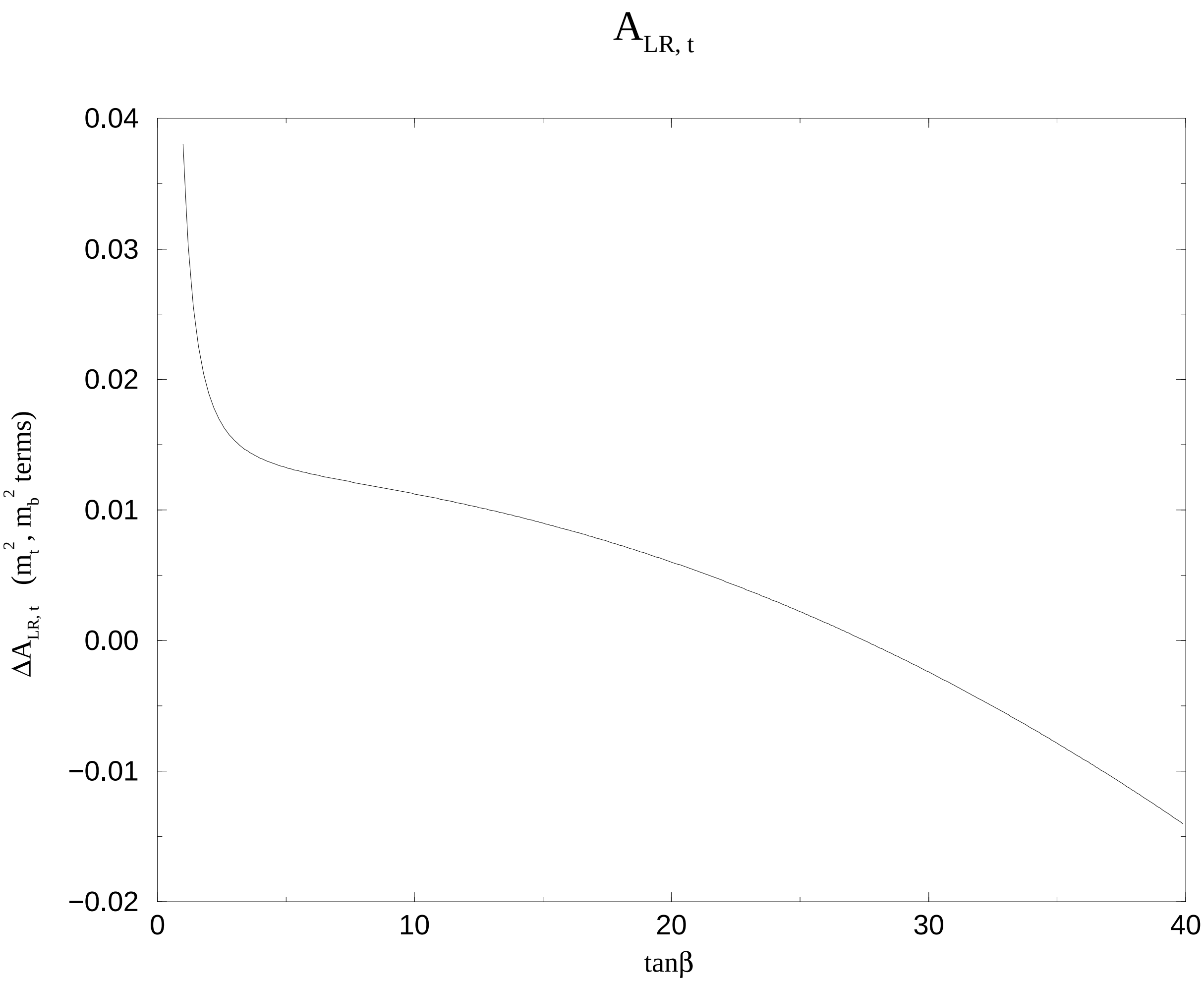

FIG. 12. Absolute effects in A_t^{LR} due to the asymptotic m_t^2 and m_b^2 logarithmic terms versus $\tan\beta$.

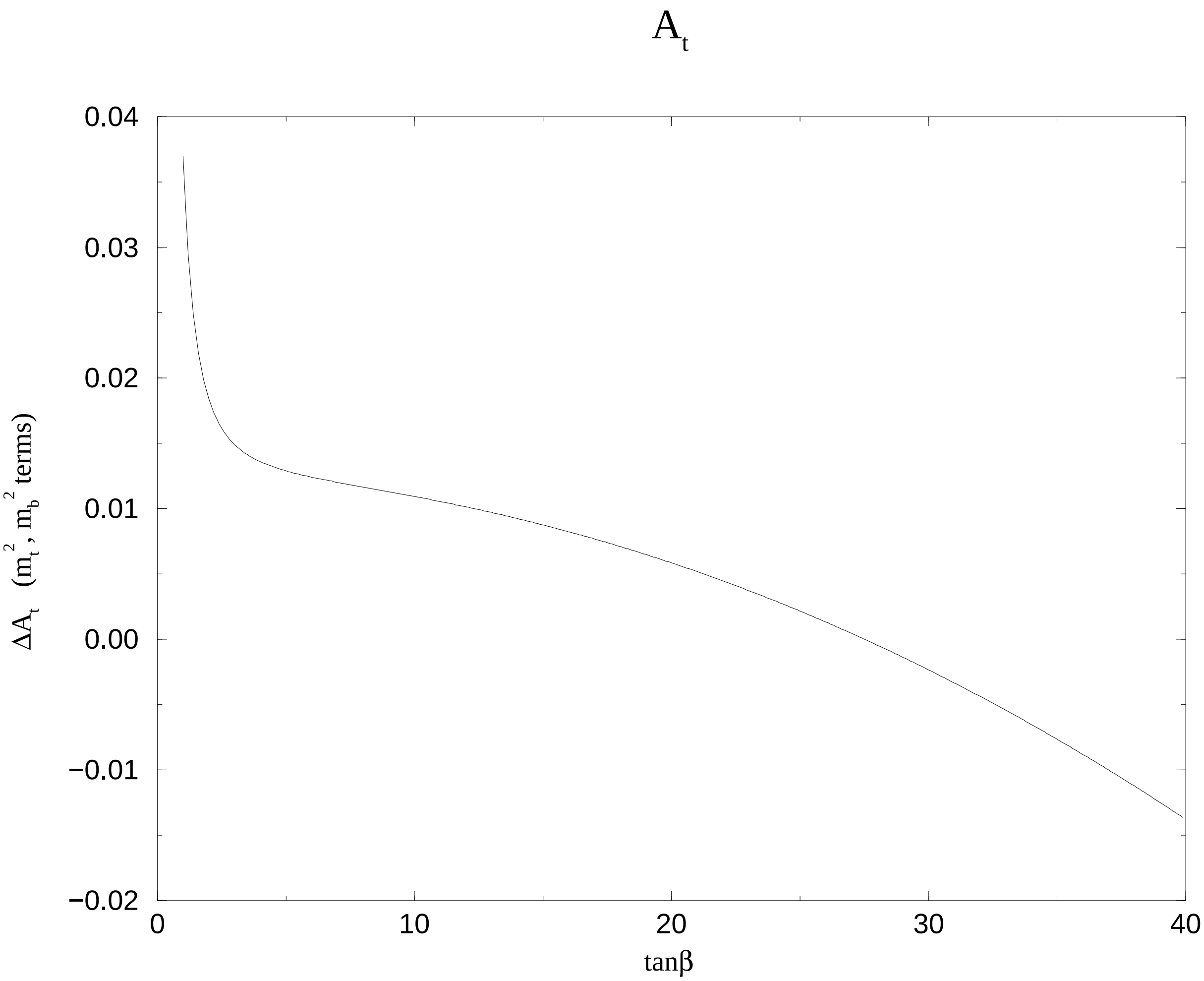

FIG. 13. Absolute effects in A_t due to the asymptotic m_t^2 and m_b^2 logarithmic terms versus $\tan\beta$.